

1 Monte Carlo Optimization for Real-Time Magnetic Domain Learning in 2 Magneto-Optical Diffractive Deep Neural Networks 3

4 Fatima Zahra Chafi^{1*}, Tomonao Matsuya¹, Hotaka Sakaguchi¹, Kanata Watanabe¹, Takayuki Ishibashi¹

5 ¹*Department of Materials Science and Bioengineering, Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka, 940-2188, Japan*
6 **e-mail: cfz@vos.nagaokaut.ac.jp*
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8 9 **Abstract**

10 We propose an optimized algorithm using Monte Carlo Method (MCM) tailored for online
11 learning in magneto-optical diffractive deep neural networks (MO-D²NN), a physical neural
12 network platform where binary-weight are determined by magneto-optical modulation of light
13 through the Faraday effect. Our derivative-free approach based MCM, iteratively adjusts the
14 magnetic domain patterns to minimize cross-entropy loss without relying on gradients at a much
15 lower computational cost. Our findings reveal that the MCM-based optimization algorithm
16 serves as a robust and viable alternative to gradient descent-based training, achieving an
17 accuracy of 96% for MNIST (Modified National Institute of Standard and Technology)
18 handwritten digits classification with only a single hidden layer, highlighting its potential as a
19 powerful approach for training MO-D²NN. We further validate its feasibility through physical
20 implementation in an experimental optical setup, confirming its practical applicability for
21 online image recognition tasks. We successfully demonstrate real-time learning of MO-D²NN
22 using the MCM algorithm.
23

24 **Keywords**

25 Monte Carlo optimization Algorithm, Magneto-Optical Diffractive Deep Neural Network,
26 Image Recognition, Online Learning.
27

28 1. Introduction

29 The pursuit of ultrafast, energy-efficient computation has spurred renewed interest in
30 physically implemented neural networks that operates beyond the limits of conventional
31 electronic systems. Diffractive optical neural networks (D²NNs) process information across
32 multiple layers by only passing light interacting with successive diffractive structures,
33 achieving high-throughput inference at minimal power [1-2]. Building on this foundation,
34 magneto-optical D²NN (MO-D²NN) harness light propagation and interference while
35 exploiting the Faraday effect in engineered magnetic thin films to encode binary weights as
36 reconfigurable magnetic domains [3-4]. This direct modulation of light through MO
37 interactions provides intrinsic parallelism and low latency, allowing real-time neural
38 computation at the speed of light. Such systems offer a compelling alternative to traditional
39 electronic accelerators for tasks ranging from image recognition to language processing, and
40 scientific computing. Despite their promise, the inability to update network parameters in real-
41 time remains a major obstacle to the widespread deployment of physical neural networks. In
42 most contemporary systems, neural networks are trained offline using numerical simulations,
43 where stochastic gradient descent (SGD) methods such as backpropagation (BP) [5], are
44 employed to minimize the loss function after which the optimized weights typically remain
45 fixed. This approach significantly prevents real-time adaptation and limits performance in
46 dynamic environments. The core limitation stems from the fundamental incompatibility
47 between conventional gradient-based algorithms and the constraints of optical and hybrid
48 neural systems. Such algorithms assume continuous, differentiable parameters, full
49 observability, and deterministic behavior; conditions rarely met in practical optics, spintronics,
50 or magneto-optics. Physical weights often require discrete or non-volatile updates, like
51 magnetization switching, which are inherently non-differentiable. This highlights the need for
52 efficient and compatible learning strategies tailored for physical implementations.

53 Monte Carlo methods (MCM) have a long legacy in physics and have demonstrated
54 remarkable versatility across diverse scientific and engineering domains. Their applications
55 extend from combinatorial optimization problems, such as scheduling and graph partitioning,
56 to neural network optimization, and reinforced learning-based decision-making systems like
57 AlphaGo [6]. They have also been widely employed within Nasa research, supporting Bayesian
58 inference and state estimation with neural fields [7,8], uncertainty quantification (UQ) in
59 aerodynamic simulations [9], and recent Bayesian/UQ-initiatives aimed at assessing spacecraft
60 performance under uncertainty and improving predictive modeling [10,11]. By employing
61 stochastic sampling with probabilistic acceptance, MCMs explore high-dimensional search
62 spaces without relying on explicit derivatives. This makes them intrinsically compatible with

63 the discrete, uncertain, and partially observable nature of physical neural systems. Their
 64 probabilistic structure also mitigates local minima, supports asynchronous updates, and
 65 minimizes unnecessary weight rewrites, key advantages for devices with limited endurance,
 66 like MO domain-switching networks. These features make MCM particularly well-suited for
 67 in-situ training, offering online adaptability and reconfigurability during operation, unlike
 68 conventional BP, which is generally limited to offline optimization and static inference.

69 This paper presents a theoretical framework of a Monte Carlo-based algorithm for online
 70 learning in MO-D²NN. The convergence behavior, classification performance on image
 71 recognition tasks (Handwritten digits, MNIST), and the underlying optimization strategy are
 72 examined through a generalized analysis. The proposed approach and its practical feasibility
 73 are confirmed were experimentally validated for online learning in MO-D²NN, as reported in
 74 our prior study [12]. The results provide new insights into Monte Carlo-driven physical neural
 75 networks and highlight its potential as a scalable scheme for next-generation neural computing.
 76 Although slower than conventional gradient-based methods in offline training, the approach is
 77 well suited to real-time physical implementation, where backpropagation cannot be directly
 78 applied. However, our research has primarily focused on MO-D²NN, where the proposed
 79 MCM-based learning has been validated, yet it is not restricted to MO-D²NN and can be
 80 extended to general diffractive deep neural networks (D²NNs). Because training relies
 81 exclusively on forward propagation combined with MCM updates, it is gradient-free and
 82 enables direct implementation in systems with discrete, or non-differentiable elements.

83 **2. Monte Carlo optimization Algorithmic Approach**

84 2.1 General MCM Framework

85 Metropolis-inspired MCM optimization algorithm employed here is a gradient-free search
 86 strategy, in which binary weights of the diffractive network are iteratively updated through
 87 stochastic proposals and loss-guided acceptance. Each trainable configuration is represented by
 88 a binary weight vector $\mathbf{v} \in \{-1, 1\}^N$, where N denotes the number of trainable domains, as
 89 diffractive elements. The optimization of these binary weights is formulated as a stochastic
 90 search process directly guided by the system loss function. At each iteration, a new
 91 configuration \mathbf{v}' is generated by flipping a predefined number of weight elements.

92 At each iteration, the optimization proceeds as follows:

- 93 - *Initialization*: The algorithm begins by randomly initializing a binary weight vector $\mathbf{v}^{(0)} \in$
 94 $\{0,1\}^N$, representing the initial magnetic domain configuration.
- 95 - *Sampling generation*: A new state \mathbf{v}' is generated by flipping a subset of entries in the current
 96 magnetic domain in \mathbf{v} , corresponding to local modifications of the domain configuration.

97 - *Loss evaluation*: The corresponding loss $L(\mathbf{v}')$ is computed by propagating the input through
 98 MO-D²NN and comparing the predicted distribution with the true label distribution. The change
 99 in loss is then obtained as:

100
$$\Delta L = L(\mathbf{v}') - L(\mathbf{v})$$

101 - *Acceptance rule*: The update rule follows a deterministic Metropolis acceptance criterion. The
 102 candidate state is deterministically accepted if $\Delta L < 0$, and the magnetic domain pattern is
 103 updated accordingly. Otherwise, the algorithm retains the previous configuration.

104 If $\Delta L < 0$ update $\mathbf{v}^{(t+1)} = \mathbf{v}'$;

105 else,

106 retain $\mathbf{v}^{(t+1)} = \mathbf{v}^{(t)}$

107 - *Iteration and Termination*: The steps of sampling, evaluation, and acceptance are repeated
 108 until convergence, or a predefined maximum number of iterations (MC-iterations) is reached,
 109 hereafter referred to as N_{max} .

110 The efficiency of MCM optimization depends on its ability to explore the weight
 111 configurations within the hidden layer, which grow exponentially with network size. At each

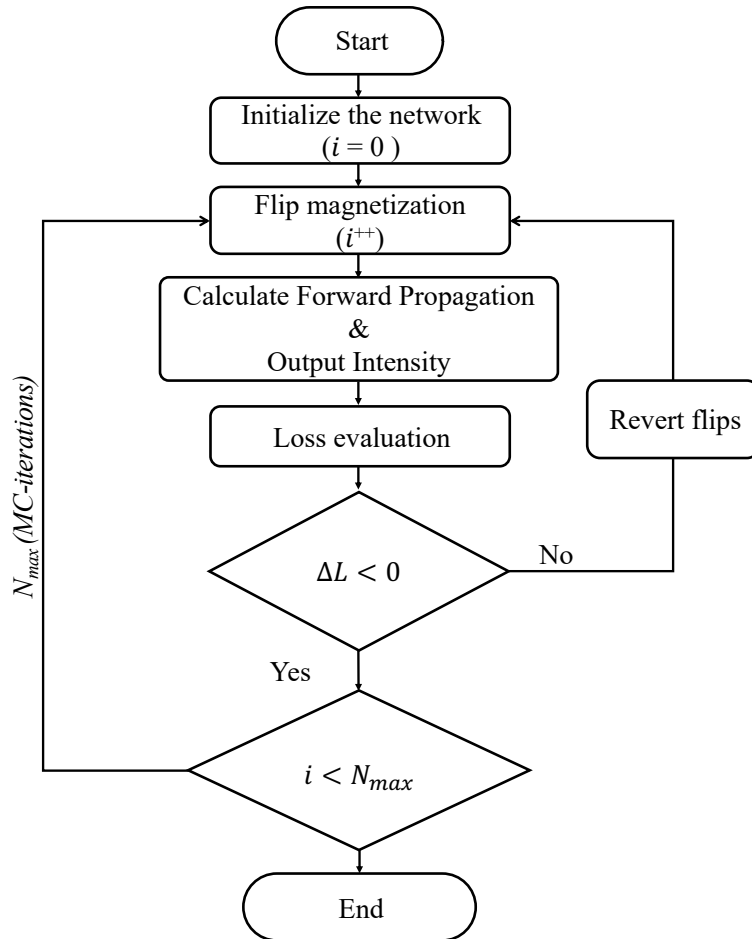


Figure 1. Algorithmic Flowchart of the employed MCM process.

112 iteration, the network processes input data, computes the loss relative to the target, proposes
 113 random domain updates in the magnetic-optical layer, and applies weight flips to generate
 114 candidate configurations aimed at reducing the loss. Candidate configurations are accepted or
 115 rejected according to a Metropolis criterion, enabling derivative-free optimization (see the
 116 conceptual flowchart, fig. 1).

117 2.2 Application of MCM in practice

118 The MCM optimization was applied to train a fully connected MO-D²NN with binary phase
 119 encoded weights for image classification tasks. The network operates as a sequence of optical
 120 transformations, where each diffractive layer modulates the incident light. The resulting
 121 intensity distribution is measured at the output plane.

122 The forward light propagation was calculated numerically, accounting for the magneto-optical
 123 (MO) effect by separately propagating the right and left circularly polarized light. The MO
 124 effect, measured as polarization rotation and ellipticity, arises from differences in refractive
 125 index and extinction coefficient for the two circular polarizations. Free-space propagation was
 126 computed using the band-limited angular spectrum method [13], derived from the Rayleigh-
 127 Sommerfield diffraction formula [14], with Fourier-based convolutions, applying band-limiting
 128 and zero-padding to maintain sampling resolution and avoid aliasing.

129 The detector plane was partitioned into ten output regions arranged in three rows (3-4-3),
 130 corresponding to digit classes (0-9). The first, second, and third rows represent digits {0-2}, {3-
 131 6}, and {7-9}, respectively. Optical intensity was evaluated within these regions at the output
 132 plane, and the predicted digit was assigned to the region exhibiting the highest intensity.
 133 Classification was therefore performed using class-specific detectors associated with the target
 134 categories. During training, the MCM-algorithm iteratively adjusts the binary configuration of
 135 the magnetic domain pattern to minimize the classification error, evaluated through the cross-
 136 entropy-loss function comparing the predicted and target intensity distributions:

$$137 \quad L = - \sum_{i=1}^C Q_i(x) \log(P_i(x; v))$$

138 where C is the number of output classes, $Q_i(x)$ is the ground-truth one-hot label vector (the
 139 true class probability distribution), and $P_i(x; v) \in [0,1]$ is the predicted class probability
 140 output for class i , under configuration v .

141 Through repeated stochastic updates and forward propagation calculations, the network
 142 converges toward a configuration that optimally maps the input to the desired output.

143 *Network architecture*

144 Calculations were performed using a single-layer MO-D²NN model shown in fig. 2,
145 comprising N trainable neurons in a 100 × 100 neuron (1 μm width). Although the proposed
146 training scheme is not restricted to a single hidden layer and has been numerically validated
147 extensive simulations beyond the single-layer configuration, involving 2, 3, 4, 5, 10 and 15
148 diffractive layers, within the same training conditions, the MCM training jointly optimizes all
149 layers simultaneously rather than sequentially optimizing each layer. However, deeper systems
150 experimentally require significantly higher alignment precision and fabrication control.
151 Therefore, for clarity and consistency with the online experimental study [12], we report
152 computations for the single-layer configuration, which provide the theoretical foundation of our
153 proposed approach. Diffractive layer was made from a perpendicular anisotropic magnetic
154 material, with two out-of-plane magnetization domains $\sigma_i = \pm 1$ (up or down), encoding phase
155 shifts of 0 or π (+1 or -1), and updated via MCM. The network was driven by linearly polarized
156 light at a wavelength of 532 nm. This wavelength was selected due to the strong magneto-
157 optical response of bismuth-gallium substituted yttrium iron garnet (BIG), which served as the
158 hidden layer in the network. BIG exhibits pronounced Faraday rotation in the visible spectral
159 range, peaking near the green region around (532 nm) [12]. As reported in our previous study
160 [18], the magneto-optical performance can be evaluated using the figure of merit (FOM),
161 defined as the ratio of Faraday rotation relative to optical absorption, which quantifies the
162 efficiency of polarization rotation to optical loss. At longer wavelength such as 633 nm, the
163 Faraday rotation decreases due to the reduced MO response, optical absorption is also lower,
164 resulting in higher transmittance and a comparable FOM. Consequently, the choice of 532 nm
165 provides an optimal balance between maximizing Faraday rotation and minimizing optical loss,
166 consistent with the intrinsic optical and MO properties of BIG films. The separations between
167 the input-to-hidden layer ($d_1 = 3.0$ mm) and between the hidden-to-output layer ($d_2 = 0.5$ mm)
168 were selected following an optimization of the inter-layer spacing, which identified 3.0 mm and
169 0.5 mm as the configurations yielding the highest accuracy. The refractive indices were set to
170 $n = 1$ for free space and $n = 2$ for the substrate. Faraday rotation angle θ_F and the ellipticity
171 η_F were assumed to be 3.3° and 0°, respectively, to maximize modulation and ensure optimal
172 interaction with the magneto-optical layer. This configuration provides strong Faraday rotation
173 while maintaining high optical transparency and stable diffraction patterns. To maintain
174 consistency, the same polarization parameters were applied in both simulations and experiment.
175 A polarizer oriented at 90° was placed between the last layer and detector, so that it was
176 perpendicular to the polarization of the incident light.

177 ***Datasets and task***

178 Classification was performed on the MNIST handwritten digits dataset, with 5,000 training
179 and 10,000 testing grayscale images of size 28×28 . Each image was rescaled and rescaled to
180 100×100 pixels. Input images were propagated through the simulated optical network, and the
181 output plane was segmented into ten-class specific detection areas corresponding to digits 0 ~
182 9, each measuring $11 \times 11 \mu\text{m}^2$. The predicted class was determined based on output plane
183 intensity, providing a direct mapping from output pixels to their expected digit.

184 ***Loss function and evaluation***

185 Convergence was tracked by loss reduction and test accuracy, with hyperparameters (flip
186 size, iterations) set from preliminary runs. Loss was evaluated as the cross-entropy between
187 target labels and normalized output intensities within each class regions, and classification
188 accuracy was computed on the test set. Binary weights were updated via MCM proposals,
189 accepting flips if loss did not increase or remained unchanged.

190 ***Training protocol***

191 Training proceeded with fixed flip-size MCM updates, described above, for up to 80,000
192 iterations. At each step, a subset of binary weights was proposed for flipping and loss was
193 evaluated for both configurations. For each configuration we report convergence curves (loss
194 vs iterations), and test accuracy vs. iterations. Optimization was entirely gradient-free, driven
195 by stochastic binary search. Backpropagation baseline was included under identical data
196 partitions for reference. When evaluating the loss, the algorithm computes ΔL , ensuring a
197 consistent loss reduction without requiring explicit gradient computation.

198 ***Implementation details***

199 Numerical simulations were conducted on a standard workstation-AI equipped with
200 NVIDIA RTX-A6000 (48 GB of graphical processing unit-GPU, a Core i9-10980XE-18 cores
201 / 36 thread of central processing unit-CPU, and 128 GB (32GBx4) of random-access memory-
202 RAM), running on Ubuntu 20.04 LTS operating system. Python (v3.10.9), NumPy (1.23.2),
203 and TensorFlow (v2.12.0) framework were used for most implementations. All experiments are
204 repeated with multiple random seeds to insure statistical robustness.

205 **2.3 Flip-size and initialization strategies in MCM optimization**

206 To further investigate the impact of the initialization states and the update strategy on the
207 optimization process, we evaluated multiple flip-sizes of the network weights. Initialization
208 plays a key role in guiding the network toward optimal configurations, while the update strategy
209 governs its exploration during each iteration. The choice of flip size shapes this process: smaller
210 flips promote finer adjustments and more stable convergence, whereas larger flips enable
211 broader exploration but may induce instability.

212 In this study, we examined two initialization configuration schemes:

213 - *Deterministic initial state*: where all the magnetic domains are initially aligned at the same
 214 direction (+1). Then number of weight flips were applied per iteration.
 215 - *Random initial state*: where domains of the network are initialized stochastically at different
 216 directions (+1 or -1), following the same flip size sequence.
 217 By systematically varying flip sizes ($F_s = \{1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, 5 \times 5, 10 \times 10 \mu m^2\}$)
 218 within the two initialization states, we assessed their impact on convergence behavior, stability,
 219 accuracy, and overall performance. This analysis reveals the influence of the initial magnetic
 220 configuration on network performance and provides a comprehensive understanding on the
 221 interplay between initialization, update strategy, and flip size on convergence and optimization
 222 efficiency, offering valuable insights for selecting parameters that ensure robust network
 223 optimization.
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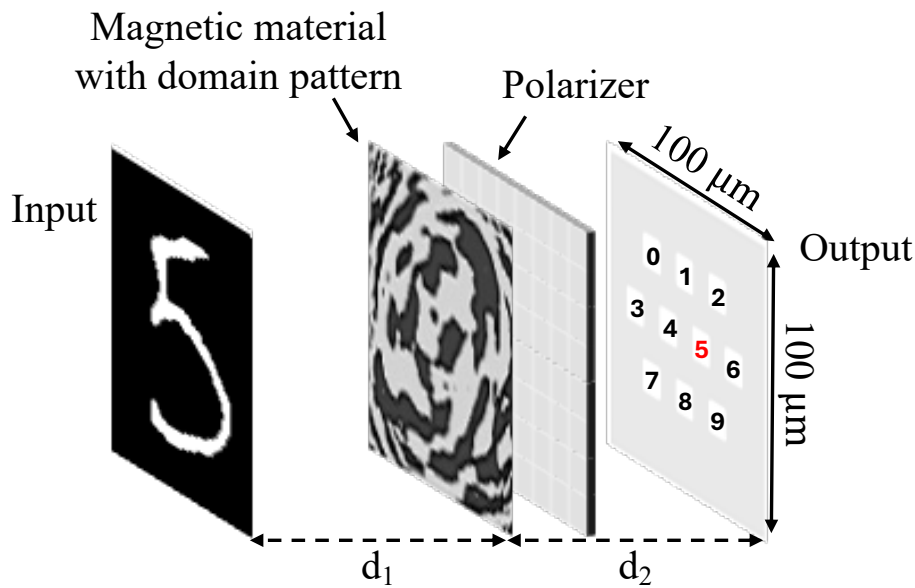


Figure 2. Structural overview of the MO-D²NN model featuring a single hidden layer and a polarizer.

225 3. Results and discussion

226 The performance of the MCM optimization was evaluated by comparing different fixed flip
 227 size strategies under different initialization configurations.

228 3.1 Effect of initialization and flip size on convergence

229 Figure 3 shows the evolution of training loss and accuracy convergence for deterministic
 230 and random initializations with a flip-size of $1 \times 1 \mu m^2$ over the same number of iterations
 231 (80,000). In both cases, the loss decreased steadily without divergence, achieving monotonic or
 232 plateau-like loss reduction, consistent with the non-worsening acceptance rule that rejects

233 unfavorable updates and allows equal-loss moves. However, their convergence characteristics
 234 differ underscoring the strong influence of initialization on MCM optimization.

235 In the case of deterministic initialization, where all weights set to +1, loss exhibits a slower
 236 initial decrease in loss, reflecting its poor starting point far from an optimal weight
 237 configuration. The acceptance rate was initially low, and the network required more iterations
 238 before reaching stable convergence. Interestingly, in most trials, the network achieved a higher
 239 final accuracy of 96%, despite showing a transient peak in the very early stage (immediately
 240 after launch), followed by a slight decrease over 10 ~ 50 iterations before resuming steady
 241 improvement. This indicates that although deterministic initialization suffers from a
 242 disadvantage at the beginning, the system can overcome its imbalance and eventually reach a
 243 strong performance.

244 Random initialization, where the weights are assigned randomly to ± 1 , shortens the
 245 computation time and yields to faster early convergence, the accuracy increases immediately,
 246 and the loss decreases without delay. In contrast, deterministic initialization requires more steps
 247 before any performance progress begins. Despite this efficiency advantage, the random scheme
 248 ultimately converges to a lower final accuracy (95%) than the deterministic one (96%).
 249 Comparing both configurations shows that each reaches a stable accuracy, with deterministic
 250 initialization converging more slowly but achieving a higher final accuracy, while random
 251 initialization accelerates the process with a modest reduction in final accuracy.

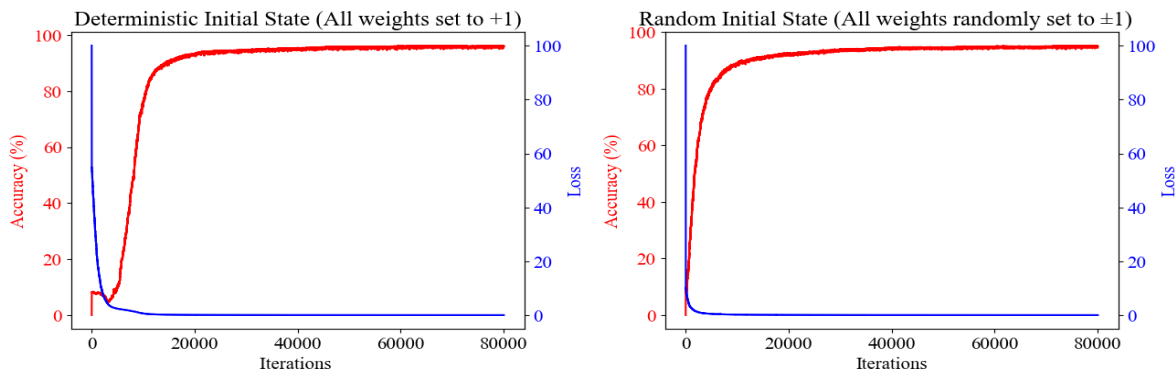


Figure 3. Classification accuracy and loss as functions of MC iterations for a fixed flip size of $1 \times 1 \mu\text{m}^2$ under deterministic and random initial state.

252 Increasing the flip size $1 \times 1 \mu\text{m}^2$ to $10 \times 10 \mu\text{m}^2$ destabilizes the optimization process,
 253 reducing accuracy and causing the loss divergence under deterministic initialization (fig. 4).
 254 The observed instability originates from excessively large domain sizes, which reduce the
 255 number of neurons. The resulting performance degradation is therefore not an intrinsic
 256 limitation of the MCM-based learning scheme, but a consequence of the underlying update
 257 process, where larger flips induce stronger perturbations while simultaneously reducing the

258 neuron counts, whereas smaller flips allow finer exploration of the configuration space and
 259 promote more stable convergence. Consequently, this behavior reflects an algorithmic trade-
 260 off between update step size, neurons number, and convergence stability, rather than a
 261 fundamental constraint of the method.

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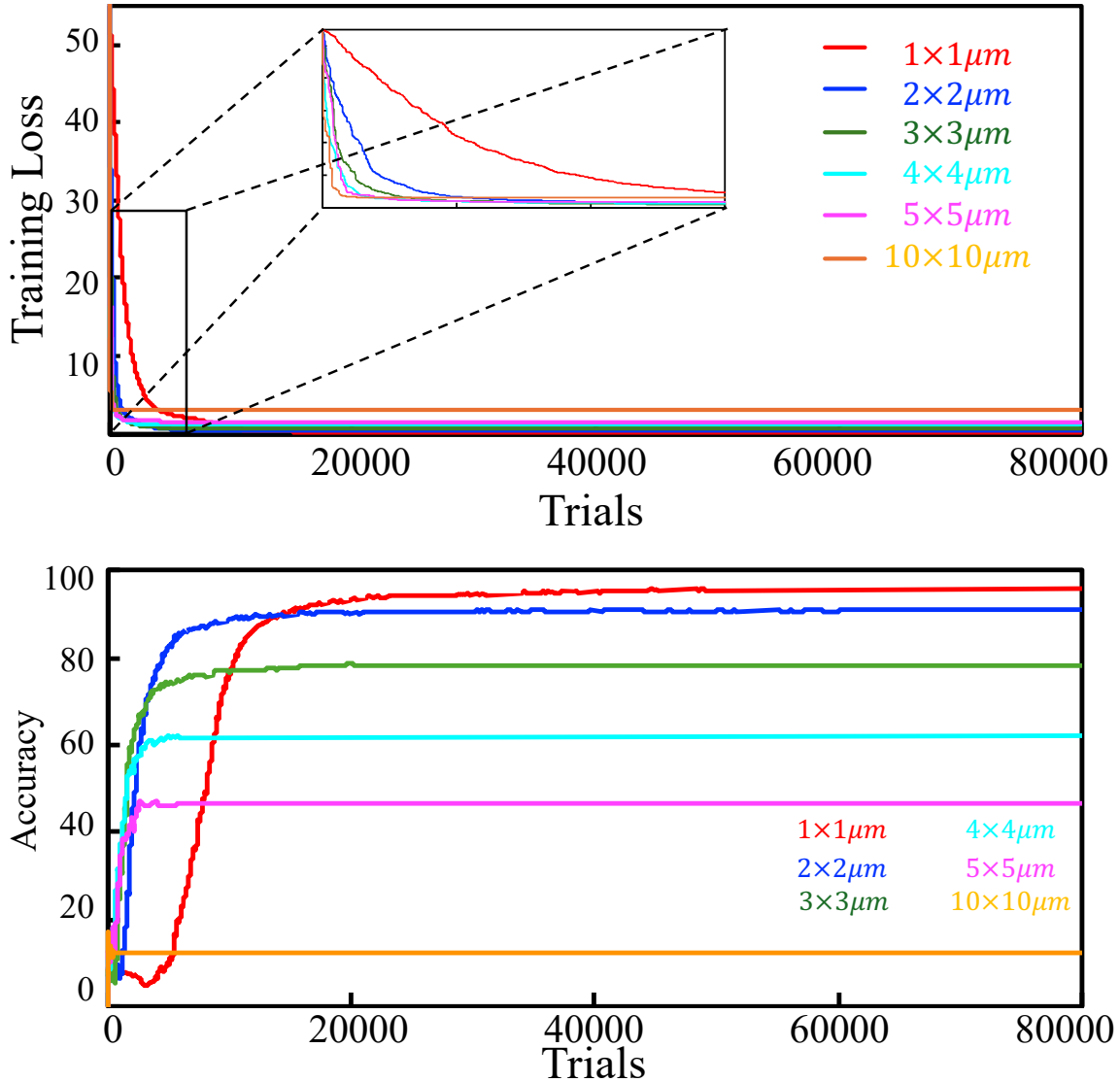


Figure 4. Training loss (top panel) and accuracy (bottom panel) versus the number of trials for various flip sizes under deterministic initialization.

263 Figure 5 displays theoretical results for a sample input, obtained after optimization from a
 264 random initial state using a flip size of $1 \times 1 \mu\text{m}^2$, and 80,000 MC iterations. The input image
 265 (a) is mapped through the trained domain distribution, revealing the magnetic domain pattern
 266 in the hidden layer (b), the output intensity (c), the masked detector-plane intensity (d), along
 267 with the corresponding classifier response. For a given input digit ‘5’, the classifier produces
 268 the highest activation for the correct label, matching the input, confirming that MCM effectively
 269 encodes input digits into distinct magnetic domain configurations for accurate digits recognition.

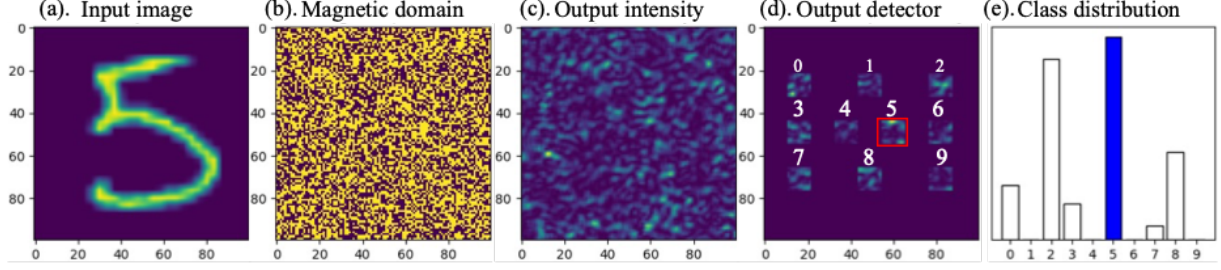


Figure 5. Simulation results of digit ‘5’ with intensity detection and true class distribution.

270 3.2 Magnetic domain evolution

271 Figures 6, 7 present the magnetic domain patterns obtained after training with different flip
 272 sizes for deterministic and random initialization. Small flip sizes (panels a-c) produce well-
 273 defined magnetic domain patterns, indicative of stable learning dynamics, whereas larger flip
 274 sizes (panels d-e) progressively degrade the pattern resolution, reflecting reduced learning
 275 stability. At a flip size of $10 \times 10 \mu\text{m}^2$, (panel f) the resolution collapses into enlarged domains,
 276 demonstrating instability driven by excessively large parameter updates. This agrees with the
 277 accuracy and loss curves, which likewise exhibit degraded performance at larger flip sizes.

278 Physically, this behavior originates from the diffraction-limited optical coupling condition,
 279 which governs light propagation between adjacent layers through its dependence on the
 280 diffraction angle (β) and the interlayer spacing. In a fully connected neural network, each
 281 neuron in one layer interacts with all neurons in the next. In an MO-D²NN, this connectivity is
 282 realized optically through the first-order diffracted light.

283 For a neuron of diameter d_n , the 1st order diffraction angle β is given by:

$$284 \quad \beta = \sin^{-1}(\lambda/2d_n)$$

285 From this angle, the interlayer distance Δ_l needed to maintain full optical connectivity
 286 followed from the geometrical separation between the farthest neuron in adjacent layers:

$$287 \quad \Delta_l = \frac{\sqrt{N_x + N_y}}{\tan\beta} d_n$$

288 where N_x and N_y are the numbers of neurons along the horizontal and vertical axes.

289 Our analysis shows that the input-to-hidden separation d_1 , evaluated over a range of values
 290 reveals $d_1 = 3.0$ mm as providing optimal classification performance. With d_1 fixed, varying the
 291 hidden-to-output spacing indicates that $d_2 = 0.5$ mm optimizes both accuracy and loss
 292 convergence. Further analysis, with d_1 fixed and d_2 set according to the fully connected equation
 293 for different flip sizes (fig. 8), showed that at non-optimal values, accuracy is only weakly
 294 affected by interlayer distance, whereas the number of neurons predominantly influences
 295 performance. These distances are consistent with diffraction-limited coupling, where first-order

296 diffracted light efficiently transmits between layers.

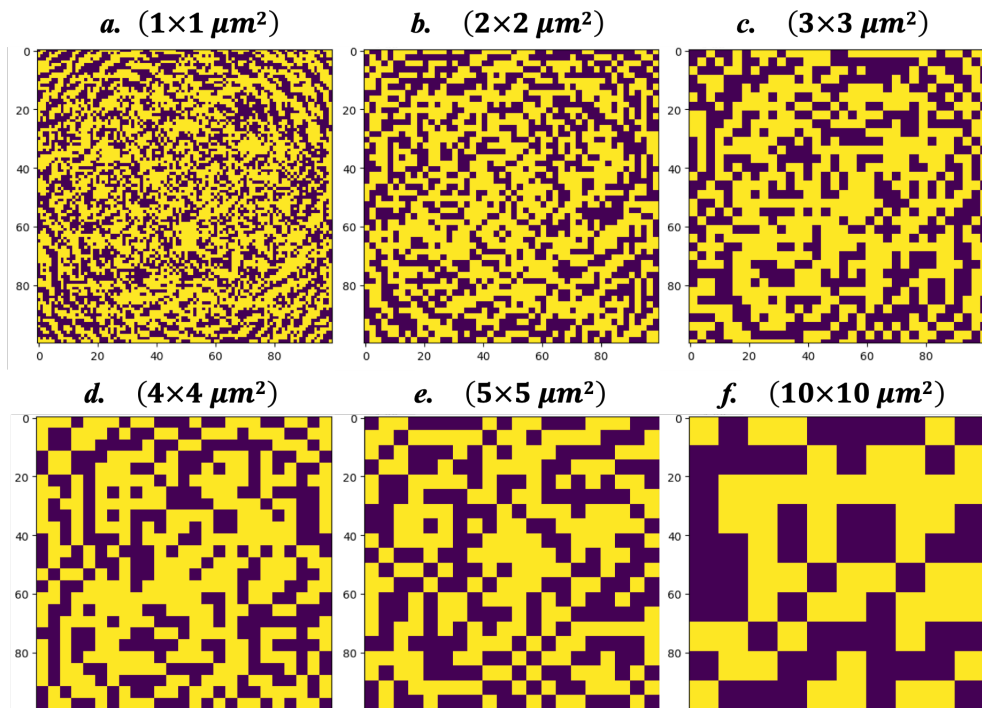


Figure 6. Magnetic domain patterns under random initial state, for flip sizes (a) 1×1 , (b) 2×2 , (c) 3×3 , (d) 4×4 , (e) 5×5 and (f) $10 \times 10 \mu\text{m}^2$.

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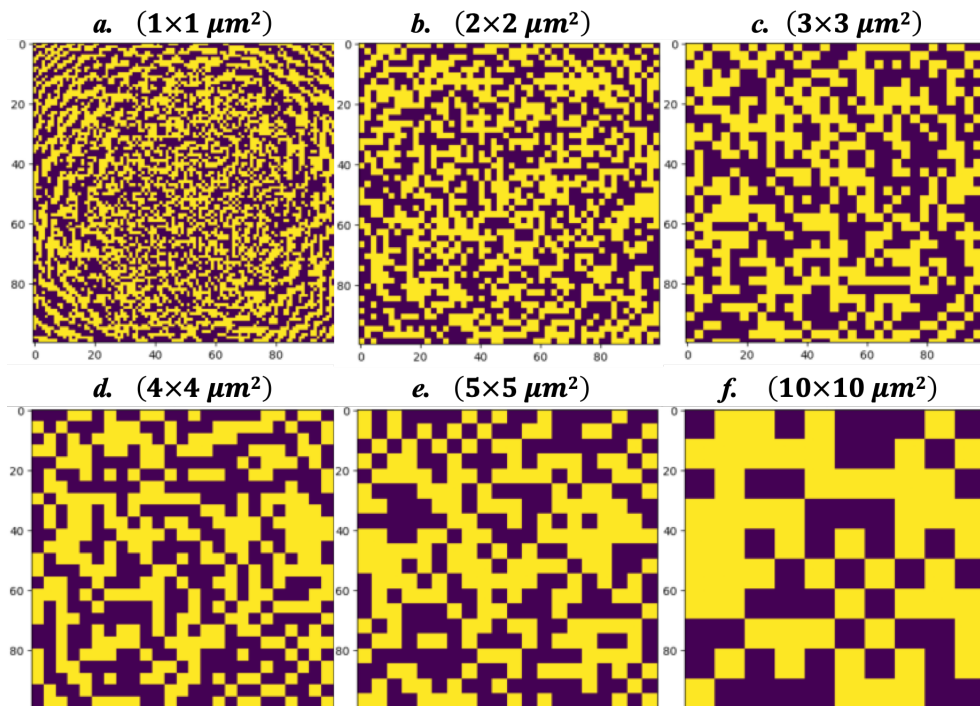


Figure 7. Magnetic domain patterns under deterministic initial state, for flip sizes: (a) 1×1 , (b) 2×2 , (c) 3×3 , (d) 4×4 , (e) 5×5 and (f) $10 \times 10 \mu\text{m}^2$.

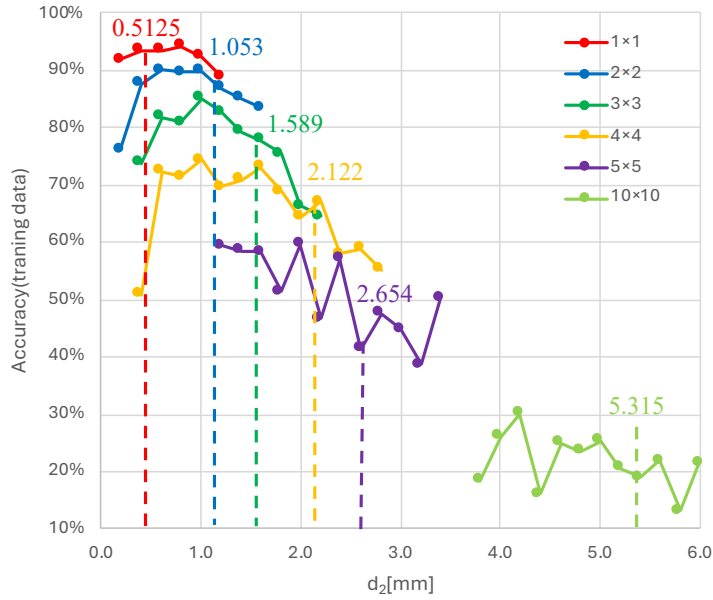


Figure 8. Accuracy as functions of the hidden-to-output layer separation d_2 for different flip sizes, with the input-hidden spacing d_1 fixed at 3.0 mm.

298 3.3 Comparative insights into MCM and SGD-based BP

299 We evaluated both MCM and SGD with BP in terms of training loss under identical
300 conditions on 5,000 training and 10,000 testing images. BP was trained for 50 epochs with a
301 batch size of 50, totaling 5,000 trials in total, while MCM used the same training and testing
302 sets over 80,000 iterations. As shown in fig. 9, MCM starts from higher initial losses, over 50
303 for deterministic initialization, and around 10 for random initial state. Whereas BP begins near
304 0.7 and converges faster per trial. Within the first trials, MCM reaches a loss profile nearly
305 identical to BP, indicating that despite its slower start, it ultimately achieves comparable
306 convergence. MCM theoretically requires more iterations ($\sim 80,000$ trials) and about 20 hours,
307 far exceeding BP, which completes the training in roughly 20 minutes.

308 Crucially, MCM permits direct online adaptation within the physical system, an approach that
309 is not accessible to BP owing to its reliance on gradient propagation and discrete activations.
310 Using our optical platform, training was performed on a single-layer MO- D^2 NN. The
311 experimental parameters were aligned with the theoretical computations outlined above,
312 including the inter-layer distances (3.0 mm, 0.5 mm) and the image size ($100 \times 100 \mu\text{m}^2$),
313 ensuring direct correspondence between simulation and implementation. Input images were
314 generated by illuminating a linearly polarized laser beam ($\lambda = 532 \text{ nm}$) onto a chromium-on-
315 glass photomask fabricated via photolithography and maskless patterning (MX-1240, Japan
316 Science Engineering Co., Ltd.), where handwritten digit patterns chemically etched. The
317 bismuth-gallium-substituted yttrium iron garnet thin film, served as the hidden-layer owing to
318 its large Faraday effect and high optical transparency. Magnetic domain was recorded using

319 thermo-magnetic recording system. Detailed fabrication and full experiment procedures are
 320 provided in our previous report [15-17, 12]. After each MCM trial, the resulting optical
 321 distribution at the output plane was recorded and classification loss was evaluated. Domain
 322 updates were retained only upon reduction of the loss, leading to progressive improvement in
 323 recognition performance over successive iterations. Processing a single MNIST image required
 324 approximately 2 hours over 200 MCM trials under the initial configuration. In an enhanced
 325 experimental arrangement presently under active development, 10 images were processed in 4
 326 hours across 1,500 trials, reflecting improved stability and throughput. Despite this progress,
 327 scaling to the full dataset remains time-intensive with the present system. Nevertheless, these
 328 findings indicate that MCM is intrinsically compatible with real-time physical implementation,
 329 whereas BP remains predominantly confined to offline computational optimization. With
 330 further optimization of the optical architecture, introducing high-speed polarized camera
 331 (HSPC) and digital micromirrors device (DMD), faster magnetic dynamic switching and
 332 improved detection, the online training in principle could be significantly accelerated to seconds
 333 or even nanoseconds, allowing 60,000 images to be processed within minutes, underscoring the
 334 potential of MCM-based optimization in physical and hybrid neuromorphic systems.

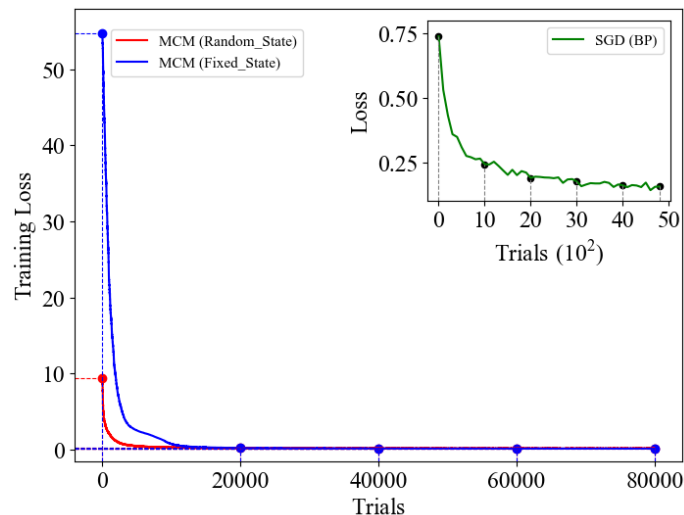


Figure 9. Loss convergence over 80,000 MCM trials for deterministic (blue) and random (red) initializations. Results are compared with a 5,000-trial BP baseline (inset, top-right).

335

336 4. Summary

337 We introduced a new training algorithm based on Monte Carlo Method (MCM) optimization
 338 for online learning in magneto-optical diffractive integrated neural network (MO-D²NN)
 339 applied to image classification. This derivative-free approach enables efficient training in
 340 discrete, non-differentiable systems. By avoiding gradient computations, the network achieved
 341 consistent 96% accuracy in classifying handwritten digits. The MCM-based algorithm was

342 successfully applied in an experimental setup, demonstrating the update of magnetic domains
343 in real time. Our study establishes theoretical and practical benefits of MCM optimization,
344 offering a scalable solution to the challenges of gradient free training. While the results are
345 promising, further refinements are needed to accelerate learning, reduce training time, improve
346 efficiency for more complex tasks, and strengthen scalability toward real-time next-generation
347 optical computing.

348 **Acknowledgements**

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350 **Conflicts of Interest**

351 The authors declare no conflict of interest.

352 **Data Availability Statement**

353 The data supporting the findings of this study can be obtained from the corresponding author
354 upon request.

355 **Author Contribution Statement**

356 Fatima Zahra Chafi conceptualized, drafted and edited the manuscript, Fatima Zahra Chafi,
357 Tomonao Matsuya, Kanata Watanabe conducted the theoretical study, Hotaka Sakaguchi
358 performed experiments, Takayuki Ishibashi reviewed and guided the analysis of the work, all
359 authors discussed the results and approved the final version.

360

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