Estimating the Absorption and Waveguiding in Porous Slabs from Multi-modal Measurements

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Knowledge of the radiation transfer through porous materials is important for a variety of engineering systems. In our experiments with layers of packed microparticles on metal, the fraction of light that is directed from normal incidence into waveguided modes is non-negligible; we seek to estimate the absorption in this porous layer with a few, simple experiments. We employ a ray transfer-function approach and provide analytic relations for the reflected, waveguided, and transmitted light. With these relations and multi-modal experimental data taken at different laser wavelengths, the interplay between reflection, waveguiding, and transmission can be estimated. We find that waveguiding is significant when the single-pass absorption of a porous slab is more than a few percent.

Keywords:

Figure 1: (a) Given the incident and reflected power, \( P_{\text{inc}} \) and \( P_{\text{ref}} \), we estimate the power that is waveguided and eventually absorbed in a porous slab and the power that is transmitted and eventually absorbed through the substrate, \( P_{\text{wav}} \) and \( P_{\text{trans}} \). (b) Block diagram of the ray-propagation model.

1 Introduction

Porous, random, and rough-surface materials are ubiquitous. Precise knowledge and characterization of their optical properties involves careful statistical analyses and often multiple samples with different optical path lengths \([1-6]\). Random surface distribution functions \([7]\), ray tracing \([8]\), linear transfer function \([9]\), Monte Carlo simulations \([10]\), and effective medium theories \([11-13]\) are some examples of the approaches that are developed to understand light propagation through random, porous, and scattering media.

In our work, we seek heuristic estimates of the light absorbed in a thin porous slab on a thick or absorbing substrate, as shown in Fig. 1(a). Such predictions offer intuition to analyze trends in a range of growing energy-related and sensing applications \([14-19]\). In recent efforts \([18, 19]\), we observe non-negligible waveguiding through porous films composed of packed microparticles.

Here, we develop a linear transfer-function model described by the block diagram in Fig. 1(b). We provide our derivation without a strong comparison to currently published experimental work; it is not
typical to study waveguiding from light at normal-incidence and it is difficult to smoothly polish and fully capture the light that exits through the porous sample from its side. However, reciprocal measurements of the normally scattered light from light through a polished, porous silicon edges indicates waveguiding can extend centimeters through a micron-thick slab [20]. We seek a self-consistent approach to quantify the power that is waveguided and transmitted from light at normal incidence.

2 Model

Figure 2: (a) Illustration of diffuse rays and waveguiding in porous slab with reflective substrate (b) Angular distribution of Gaussian input, scattering function $S(k)$, and distributions $Y(k)$ after propagation distances $z = 2d$ and $z = 4d$ after 1 and 2 reflections. The power of the fields that undergo total internal reflection (TIR) is determined by an angular $k$-cutoff. (c) Angular measurements of wavelength-dependent $S(k)$. (d) Integrating sphere measurements of wavelength-dependent $P_{\text{ref}}/P_{\text{inc}}$

The linear transfer-function model [Fig. 1(b)] shows the operations for an angular distribution of rays or $k$-space. The top row of operations represents one round trip through the slab starting from the air-slab interface. The angular distribution of rays is convolved by the scattering angular transfer function $S(k)$ when it travels through the slab. $S(k)$ represents a distribution of rays after traveling a distance $z = d$ in the porous slab. The diverging rays are illustrated in [Fig. 2(a)].

The model is agnostic to the exact scattering function $S(k)$, which can be Gaussian or Lorentzian with a long tail [4]. The distribution of rays in the linear system is described by self-convolutions of the input and scattering function. A Heaviside function represents a cut-off angle at the slab-air interface; above a threshold $|k|$ the rays undergo total internal reflection (TIR). Figure 2(b) shows an input Gaussian with Lorentzian $S(k)$, and outputs $Y(k)$ after travel distances $z = 2d$ and $z = 4d$.

The equations for $P_{\text{ref}}$, $P_{\text{wav}}$, and $P_{\text{abs}}$ are derived from the geometric series of thin film interference equations and the block diagram [Fig. 1(b)]:

$$\frac{P_{\text{ref}}}{P_{\text{inc}}} = R_{\lambda} + \frac{F_{\lambda}T_{\lambda}^2(1 - A_w)^2(1 - A_m)}{1 - R_{\lambda}(1 - A_w)^2(1 - A_m)} \quad (1)$$

$$\frac{P_{\text{wav}}}{P_{\text{inc}}} = \frac{F_{\lambda}T_{\lambda}(1 - A_w)A_m}{1 - R_{\lambda}(1 - A_w)^2(1 - A_m)} + \frac{(1 - F_{\lambda})T_{\lambda}(1 - A_w)A_m}{1 - (1 - A_w)^2(1 - A_m)} \quad (2)$$

$$\frac{P_{\text{abs}}}{P_{\text{inc}}} = \frac{F_{\lambda}T_{\lambda}(1 + (1 - A_w)(1 - A_m))A_w}{1 - R_{\lambda}(1 - A_w)^2(1 - A_m)} + \frac{(1 - F_{\lambda})T_{\lambda}(1 + (1 - A_w)(1 - A_m))A_w}{1 - (1 - A_w)^2(1 - A_m)} \quad (3)$$

where $R_{\lambda}$ is the average fraction of light that is reflected at the air-slab or slab-air interface, $T_{\lambda} = 1 - R_{\lambda}$, $F_{\lambda}$ is the fraction of $P_{\text{inc}}$ that does not experience TIR (and $1 - F_{\lambda}$ is the fraction of light that undergoes TIR), $A_w$ is the average absorption over the optical path of the slab (and $1 - A_w$ is the light fraction transmitted through one optical pass of the slab), and $A_m$ is the average absorption at the substrate.
(and $1 - A_m$ is the light fraction that is reflected at the substrate). We vary values of $A_w$ relate to varying-thickness porous layers: $A_w \sim 1 - \exp(-\alpha z)$ where $\alpha$ is the absorption coefficient, and $z$ is the mean free path through the film or slab. The absorbed power in the porous slab increases significantly when the slab thickness increases and when the substrate is more reflective. These equations represent simple cases where the reflection and absorption coefficients are averaged and do not depend on $k$.

The reflected power ratio has two contributions: light reflected at the air-slab interface and light that does not undergo TIR once transmitted into the slab. Meanwhile, the power in the waveguide and substrate carries separate contributions whether or not the ray undergoes TIR. The numerators in Eqs. 1-3 represent the fraction of power at the first junction where $P_{ref}$, $P_{wav}$, and $P_{trans}$ are first drawn in the diagram [Fig. 1(d)]. The form of the denominators is from a geometric-series sum, $\sum_{n=0}^{\infty} s^n = (1 - s)^{-1}$, where $s = R_\lambda (1 - A_w)^2 (1 - A_m)$ and $s = (1 - A_w)^2 (1 - A_m)$ are the transmission coefficients after one round trip without TIR and with TIR. Finally, it can be shown that $P_{ref} + P_{wav} + P_{trans} = P_{inc}$.

To solve for $P_{wav}/P_{inc}$ and $P_{trans}/P_{inc}$ from experimental data, it is necessary to solve Eq. 1 for $R_\lambda$ and $F_\lambda$, which we propose one could do with data at different wavelengths. First, one determines the relative wavelength dependence for $F_\lambda$ with an angular distribution measurement $S(k)$ [Fig. 2(c)]. Then, an integrating sphere measurement [Fig. 2(d)] provides $P_{ref}/P_{inc}$. Finally, to account for the uneven surface porosity, $R_\lambda$ may scale with the Fresnel reflectivity measurement, which also depends on wavelength. With reliable tuning of the absorption ($A_w$, $A_m$) at different wavelengths and mean free path $z$, solutions for $R_\lambda$ and $F_\lambda$, are straightforward, and are then used to calculate $P_{wav}$ and $P_{trans}$. Other approaches to solving a linear set of equations with additional wavelength measurements could fit additional unknowns and enable higher degrees of precision.

3 Demonstration and Conclusion

We illustrate Eqs. 1-3 in Fig. 3. The range of calculated values is smaller for more absorbing or thicker porous slabs (larger $A_w$) and more reflecting substrates (smaller $A_m$), which indicates where our model may be more useful. Notably, if the slab absorption $A_w$ is more than a few percent, then the light is absorbed by the porous slab is greater than that absorbed by the substrate.

In conclusion, we present a ray transfer-function approach to analyze the reflected, waveguided, and transmitted light from a porous slab on a reflecting substrate. Our approach is valuable since it is difficult to accurately measure the light that is waveguided and absorbed in the porous slab. The analytic expressions for the absorbed power in the slab can be fit with experimental measurements at different wavelengths.

Figure 3: Results of Eqs. 1-3 for (a) $F_\lambda = 0.1, R_\lambda = 0.1$, (b) $F_\lambda = 0.8, R_\lambda = 0.2$, (c) $F_\lambda = 0.5, R_\lambda = 0.5$, and (d) $F = 0.5, R_\lambda = 0.8$. 

3
4 Conflicts of Interest
The authors declare no conflicts of interest.

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References


