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Uncertainty analysis of spectral flux measurement using Monte Carlo simulation

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Abstract. In photometry, spectrally integrated quantities are commonly used, and their uncertainties are calculated using classical approaches. Since determining correlations at different wavelengths and effects on spectrally integrated quantities is rather complex, a noise modification of the spectral value using Monte Carlo simulation was applied to estimate unknown correlations in the uncertainty of total luminous flux based on integrated spectral luminous flux values. For this aim, an LED with 6500 K correlated colour temperature was measured in an integrated sphere flux measurement system. The correlations between the measurements at different wavelengths were analysed, and the uncertainty boundaries of the integrated quantity and total luminous flux were obtained.

Keywords: Uncertainty, Monte Carlo, Base functions, Integrated quantities, Correlations between quantities at different wavelengths.

1 Introduction

Photometric parameters like total luminous flux, colour coordinates, photometer responsivity, spectral mismatch correction factor, etc., are attained by spectrally integrating quantities. The guide to the expression of uncertainty in measurement (GUM) uncertainty framework or Monte Carlo (MC) simulations [1–3] are commonly used to assign uncertainties of the parameters at different wavelengths in spectral measurements. However, fully uncorrelated contributions will average out while integrating the quantities over wavelength, and the fully correlated contributions can be assigned to the integrated value without any change. The question is: what happens with partly correlated quantities?

The correlation between the spectral values of photometric parameters at various wavelengths possesses considerable intricacy. To overcome the challenges involved in determining the correlation between values at different wavelengths, Kärhä, in 2017 [4], introduced an innovative MC based technique.

In this study, the novel MC-based method is utilised for analysing the uncertainty of the total luminous flux, one of the photometric integrated quantities.

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2 Theoretical framework

The luminous flux is one of the main quantities of the light sources. This quantity value is determined using several methods and systems based on spatial and/or spectral measurements. The total luminous flux can be determined depending on the measurement system and the integration of spatial and/or spectral measurement values.

As the luminous flux measurement, spectral measurements are generally preferred for more accurate results. To obtain the total luminous flux value, the spectral measurement results are integrated over the visible spectrum (typically from approximately 380 nm to 780 nm).

The relation between the spectral measurements at different wavelength positions is found to be rather complex. The fully uncorrelated contributions will average out while integrating the quantities over wavelength, and the fully correlated contributions can be assigned to the integrated value without any change. But the calculation of the partial correlations is very difficult.

Since there is a great difficulty in calculating the correlation between values at different wavelengths, a noise modification of the spectral value using the novel MC simulation was applied and used for analysing and estimating the possible effects of partly correlated uncertainties in

Table 1. Uncertainty budget of the radiant flux of the lamp at several wavelengths.

Source of uncertainty	350 nm	400 nm	500 nm	600 nm	700 nm	Unit
Reference lamp value	1.80×10^{-22}	1.96×10^{-20}	3.55×10^{-16}	1.87×10^{-15}	4.50×10^{-17}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Reference lamp drift	1.70×10^{-9}	2.46×10^{-9}	2.25×10^{-7}	6.75×10^{-8}	3.56×10^{-10}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Repeatability of reference lamp	1.56×10^{-11}	1.39×10^{-10}	7.37×10^{-8}	6.94×10^{-8}	5.72×10^{-10}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Repeatability of the lamp measured	8.58×10^{-11}	1.34×10^{-10}	1.71×10^{-9}	6.43×10^{-10}	9.72×10^{-11}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Repeatability of aux. lamp when ref. lamp is in sphere	7.94×10^{-12}	1.48×10^{-11}	1.30×10^{-9}	3.80×10^{-10}	6.07×10^{-12}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Repeatability of aux. lamp when the lamp measured is in sphere	3.02×10^{-11}	1.97×10^{-11}	3.24×10^{-9}	4.03×10^{-10}	4.22×10^{-12}	$(\mathrm{W{\cdot}nm^{-1}})^2$
The current of reference lamp	2.74×10^{-16}	2.74×10^{-16}	2.74×10^{-16}	2.74×10^{-16}	2.74×10^{-16}	$(\mathrm{W{\cdot}nm^{-1}})^2$
The current of the lamp	6.77×10^{-16}	6.77×10^{-16}	6.77×10^{-16}	6.77×10^{-16}	6.77×10^{-16}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Linearity of spectrometer	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Wavelength accuracy of the spectrometer	1.05×10^{-14}	1.05×10^{-14}	1.05×10^{-14}	1.05×10^{-14}	1.05×10^{-14}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Straylight of the spectrometer	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Spatiality sensitivity of the spectrometer	5.03×10^{-13}	5.03×10^{-13}	5.03×10^{-13}	5.03×10^{-13}	5.03×10^{-13}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Ambient sensitivity of the spectrometer	9.52×10^{-14}	9.52×10^{-14}	9.52×10^{-14}	9.52×10^{-14}	9.52×10^{-14}	$(\mathrm{W{\cdot}nm^{-1}})^2$
Uncertainty ($k = 2$)	8.59×10^{-5}	1.05×10^{-4}	1.10×10^{-3}	7.44×10^{-4}	6.44×10^{-5}	$(\mathrm{W{\cdot}nm^{-1}})$

the measurement of spectral flux data to spectrally integrated total luminous flux quantity. A series of orthogonal base functions were used in this MC-based method to simulate potential systematic deviations.

The uncertainty of spectral radiant flux at each wavelength, expressed in equation (1) and the uncertainty of the total luminous flux, expressed in equation (2) were calculated using classical approaches GUM framework without any correlation contribution.

The uncertainty sources of the spectral radiant flux are given in Table 1.

$$\phi_{\mathrm{eT}}(\lambda) = \phi_{\mathrm{eR}}(\lambda) \cdot \left(\frac{y_{\mathrm{T}}(\lambda)}{y_{\mathrm{R}}(\lambda)}\right) \cdot \left(\frac{y_{\mathrm{AR}}(\lambda)}{y_{\mathrm{AT}}(\lambda)}\right) \tag{1}$$

$\Phi_{\mathrm{eT}}(\lambda)$: spectral radiant flux values of the measured lamp,
$\Phi_{\mathrm{eR}}(\lambda)$: spectral radiant flux values of the reference lamp,
$y_{\mathrm{T}}(\lambda)$: spectral measured values when the measured lamp is on,
$y_{\mathrm{R}}(\lambda)$: spectral measured values when the reference lamp is on,
$y_{\mathrm{AT}}(\lambda)$: spectral measured values when the measured lamp is pleased in the integrating sphere and only the auxiliary lamp is on,
$y_{\mathrm{AR}}(\lambda)$: spectral measured values when the reference lamp is pleased in the integrating sphere and only the auxiliary lamp is on,

$$\phi_{\mathrm{T}} = K_{\mathrm{m}} \cdot \int_{380\,\mathrm{nm}}^{780\,\mathrm{nm}} V(\lambda) \cdot \phi_{\mathrm{eT}}(\lambda) \cdot \mathrm{d}\lambda \tag{2}$$

Φ_{T}: total luminous flux value of measured lamp,
$\Phi_{\mathrm{eT}}(\lambda)$: spectral radiant flux values of the measured lamp,
$V(\lambda)$: luminous efficiency function,
K_{m}: maximum spectral luminous efficacy for photopic vision.

Later, the novel MC-based method was applied to the spectral values. The novel MC-based method has been employed to account for the correlations at different wavelengths for the total luminous flux uncertainty analysis.

For this aim, the spectral radiant flux values, $\Phi_{\mathrm{eT}}(\lambda)$, undergo a modification process in equation (3) by adding a component based on an uncertainty-weighted random error function, given in equation (4). The error function includes the number of base functions and the weight as given in equation (5) and equation (6), respectively.

$$\phi_{\mathrm{eTn}}(\lambda) = \phi_{\mathrm{eT}}(\lambda) \cdot (1 + \delta(\lambda) \cdot u_{\mathrm{c}}(\lambda)) \tag{3}$$

$$\delta(\lambda) = \sum_{i=0}^{N} \gamma_i \cdot f_i(\lambda) \tag{4}$$

$$f_i(\lambda) = \sqrt{2} \cdot \sin\left[i \cdot \left(2\pi \cdot \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1}\right) + \phi_i\right] \tag{5}$$

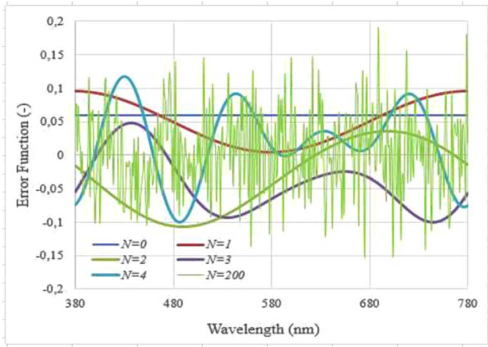

Fig. 1. Error functions for some N values.

$$\{\gamma_0, \gamma_1, \ldots, \gamma_N\} = \left\{ \frac{Y_0}{\sqrt{Y_0^2 + Y_1^2 + \ldots + Y_N^2}}, \frac{Y_1}{\sqrt{Y_0^2 + Y_1^2 + \ldots + Y_N^2}}, \ldots, \frac{Y_N}{\sqrt{Y_0^2 + Y_1^2 + \ldots + Y_N^2}} \right\} \quad (6)$$

$\Phi_{\mathrm{eTn}}(\lambda)$: spectral radiant flux with the random error function,
$u_{\mathrm{c}}(\lambda)$: combined uncertainty of spectral radiant flux,
$\delta(\lambda)$: random error function,
γ_i: weight function,
$f_i(\lambda)$: base function,
Φ_i: phase,
λ_1 and λ_2: start and end wavelength of the range,
Y_i: random variables,
N: number of weight and base functions

The Y_i variables, which relate to weighting the base functions, and the Φ_i phase terms, which correspond to base functions, are randomly generated with 20,000 iterations. Following each random generation of these variables, the resultant modified radiant flux values are computed for every wavelength in equation (3). The $f_0(\lambda)$ function is assumed to be equal to one as a special case to have the full correlation. Several random error functions given in equation (4) are illustrated in Figure 1 to show their pattern.

3 Modification and results

An LED light source was measured using the integrating sphere flux measurement system with a spectrometer. Using the substitution method, the spectral radiant flux values of the LED light source were calculated in equation (1) for each wavelenth. Then integrating the spectral radiant flux values in equation (2), the total luminous flux was obtained. The uncertainty of total luminous flux, expressed in equation (2), was calculated using classical approaches GUM framework as 0.69%.

For the novel MC-based method for uncertainty analysis, a series of codes were developed in Python and are mainly based on the modules shared in empir19nrm02 GitHub repository [5, 6].

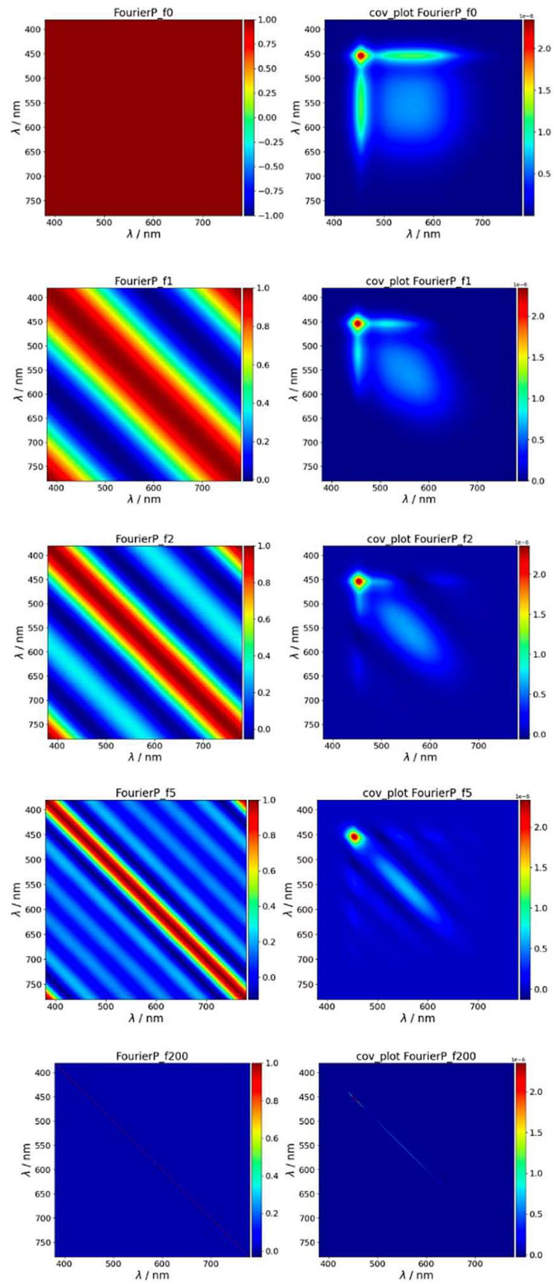

Fig. 2. Correlation (on the left) and covariance (on the right) matrices for N given in ascending order.

To determine the unpredictable uncertainty boundaries of the total luminous flux in equation (2), the spectral radiant flux values were disturbed using the MC-based method [1]. Total luminous flux was obtained by integrating the disturbed spectral radiant flux in equation (2).

The modification process was repeated for each of the N base function numbers. After each modification, the correlations between the values at different wavelengths were calculated, and the covariance and correlation matrices for $N = 0$, $N = 1$, $N = 2$, $N = 5$, and $N = 200$ are given in Figure 2.

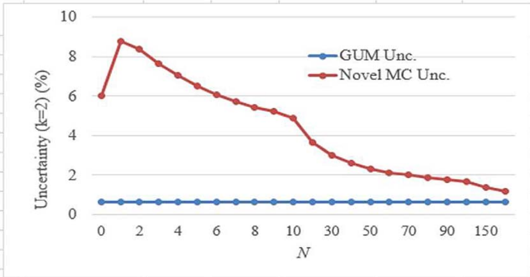

Fig. 3. Uncertainty ($k = 2$) of the total luminous flux.

The case of full correlation, in a special case where N equals zero and the maximum effect of correlation where N equals one are seen in Figure 2.

The other matrices are displayed in Figure 2 for three different values of N, exemplary $N = 2$, $N = 5$, and $N = 200$, to facilitate a comparative analysis. The correlation matrices with $N = 0$ and $N = 1$ can be compared to the correlation matrices for these N values. Figure 2 illustrates a diminishing trend in correlations as the value of N increases.

The uncertainties associated with the total luminous flux were also depicted in Figure 3 versus N number after the modification.

The boundaries of the possible uncertainties, which cannot be unpredicted and determined, are discernible in Figure 3. The maximum uncertainty is observed when N attains unity. The convergence of total luminous flux uncertainty towards the calculated uncertainty value from the GUM framework is observed as N increases.

4 Conclusions

The total luminous flux of an LED light source with 6500 K CCT, a photometric integrating parameter, was attained by integrating the spectral radiant flux. The uncertainty calculation of the total flux using GUM framework [2] resulted in a value of 0.69%, which reflects the combined effects of various sources of uncertainty. However, the GUM analysis did not take into consideration any correlations that may exist between the different sources of uncertainty.

Since there is complexity in calculating the correlation between values at different wavelengths, a novel MC-based method was utilised for analysing and estimating the 0uncertainties of the spectrally integrated total luminous flux in this study. And the possible uncertainty distributions were attained. The maximum uncertainty is obtained 8.76% while GUM is 0.69%.

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Conflicts of interest

The authors declare that they have no competing interests to report.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors on request.

Author contribution statement

Şenel Yaran; writing – original draft preparation/writing – review and editing/investigation/methodology. Zühal Alpaslan Kösemen; writing – review and editing/software/investigation. Çağrı Kaan Akkan; investigation. Hilal Fatmagül Nişancı; software. Udo Krüger; supervision/software/conceptualization/methodology. Armin Sperlin; supervision/methodology.

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