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RESEARCH

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Dimensionality of random light fields

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Abstract

Background: The spectral polarization state and dimensionality of random light are important concepts in modern optical physics and photonics.

Methods: By use of space-frequency domain coherence theory, we establish a rigorous classification for the electricfield vector to oscillate in one, two, or three spatial dimensions.

Results: We also introduce a new measure, the polarimetric dimension, to quantify the dimensional character of light. The formalism is utilized to show that polarized three-dimensional light does not exist, while an evanescent wave generated in total internal reflection generally is a genuine three-dimensional light field.

Conclusions: The framework we construct advances the polarization theory of random light and it could be beneficial for near-field optics and polarization-sensitive applications involving complex-structured light fields.

Keywords: Dimensionality, Polarization, Random light

Background

Polarization is a fundamental property of light [1, 2], specified by the orientation of the electric-field vector. In a particular coordinate system, the electric component of random light may fluctuate in three orthogonal spatial directions, but by rotating the reference frame it may turn out that the field vector actually is restricted to a plane, or even that it fluctuates in just a single direction. Optical fields can thereby be classified into one-dimensional (1D), two-dimensional (2D), or three-dimensional (3D) light, depending on the minimum number of orthogonal coordinate axes required to represent them. The dimensional nature of light plays an essential role in addressing polarization characteristics of complex-structured light fields, e.g., electromagnetic near and surface fields [3–5] as well as tightly focused optical beams [6–9], which are frequently exploited in near-field probing [10], single-molecule detection [11], particle trapping [12], among other polarization-sensitive applications. Yet, no systematic theory has so far been developed which provides rigorous means to categorize and to characterize the dimensionality of light.

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In this work, we consider the dimensionality of random light fields and show that the number of nonzero eigenvalues of the real part of the 3×3 polarization matrix provides the required information for such a dimensional classification. We also establish a quantitative measure, the spectral polarimetric dimension, describing the intensity-distribution spread or the 'effective' dimensionality of a light field. The general formalism is utilized to demonstrate that polarized 3D light does not exist, while a partially polarized evanescent wave created in total internal reflection is unambiguously a genuine 3D light field.

Methods

The polarization properties of a random light field are in the space–frequency domain described by the spectral polarization matrix [1, 2, 13, 14]

$$\boldsymbol{\Phi}(\mathbf{r},\omega) = \left\langle \mathbf{E}^*(\mathbf{r},\omega)\mathbf{E}^{\mathrm{T}}(\mathbf{r},\omega)\right\rangle. \quad (1)$$

In the stationary case the generally three-component column vector $\mathbf{E}(\mathbf{r},\omega)$ is a realization representing the electric field at point $\mathbf{r}$ and (angular) frequency ω, whereas in the nonstationary case it is the Fourier transform of the space–time domain field. In addition, the angle brackets, asterisk, and superscript T denote ensemble averaging, complex conjugation, and matrix transpose, respectively. Alternatively, the spectral polarization matrix can

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be introduced via the generalized Wiener–Khintchine theorem [14, 15]. As the polarization matrix is Hermitian and nonnegative definite, we can express it as

$$\boldsymbol{\Phi}(\mathbf{r},\omega) = \boldsymbol{\Phi}'(\mathbf{r},\omega) + i\boldsymbol{\Phi}''(\mathbf{r},\omega), \tag{2}$$

where the real part $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ is a symmetric and positive semidefinite matrix, while the imaginary part $\boldsymbol{\Phi}''(\mathbf{r},\omega)$ is skew symmetric. The three eigenvalues of $\boldsymbol{\Phi}(\mathbf{r},\omega)$ are real and nonnegative.

Let us next consider a situation that $\boldsymbol{\Phi}(\mathbf{r},\omega)$ is subjected to an orthogonal transformation. Such an operation is represented by a real-valued 3×3 matrix $\mathbf{Q}$ which obeys $\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}^{-1}$ and $\det \mathbf{Q} = 1$. The matrix $\mathbf{Q}$ can be identified with a rotation of the Cartesian reference frame, implemented by three successive Euler rotations about the Cartesian axes. Unlike for unitary transformations in general, the physical polarization state of the field does not change in an orthogonal transformation, although its mathematical representation does. In particular, because the real part $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ is symmetric, it can be diagonalized by a specific orthogonal transformation that we denote by $\mathbf{Q}_0$. In this intrinsic coordinate frame, the polarization matrix reads as

$$\boldsymbol{\Phi}_0(\mathbf{r},\omega) = \mathbf{Q}_0^{\mathrm{T}}\boldsymbol{\Phi}(\mathbf{r},\omega)\mathbf{Q}_0 = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} + i \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}, \tag{3}$$

where the eigenvalues $a_1 \geq a_2 \geq a_3 \geq 0$ of $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ are the principal intensities and the vector $\mathbf{n} = (n_1, n_2, n_3)$ is the angular-momentum vector [16–18]. In addition, a_1 is the largest while a_3 is the smallest diagonal element of $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ that can be obtained by an orthogonal transformation [19].

Results and discussion

Dimensionality of random light

In the case that only one eigenvalue of the real part $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ is nonzero, the electric-field vector fluctuates in just a single direction and the light is considered one dimensional. If, instead, only one eigenvalue of $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ is zero, the electric field is restricted to a plane and the light is regarded two dimensional. However, when every eigenvalue is positive, the intensity of each Cartesian field component is nonzero for any orientation of the frame (since a_3 is the smallest obtainable intensity along a coordinate axis) and the electric-field vector fluctuates in all three dimensions. The physical dimensionality of the light field is thereby determined by the eigenvalues of $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ as

$$\text{1D light:} \quad a_1 > 0,\ a_2 = 0,\ a_3 = 0; \tag{4}$$

$$\text{2D light:} \quad a_1 > 0,\ a_2 > 0,\ a_3 = 0; \tag{5}$$

$$\text{3D light:} \quad a_1 > 0,\ a_2 > 0,\ a_3 > 0. \tag{6}$$

We further define isotropic 2D light as one for which $a_1 = a_2$ in Eq. (5) and isotropic 3D light as one that satisfies $a_1 = a_2 = a_3$ in Eq. (6). In particular, because $\det \boldsymbol{\Phi}'(\mathbf{r},\omega) = a_1 a_2 a_3$ is invariant under orthogonal transformations, Eqs. (4)–(6) imply that both 1D and 2D light obey $\det \boldsymbol{\Phi}'(\mathbf{r},\omega) = 0$, while for a genuine 3D light field $\det \boldsymbol{\Phi}'(\mathbf{r},\omega) > 0$.

We stress that the number of nonnegative eigenvalues of the full complex polarization matrix $\boldsymbol{\Phi}(\mathbf{r},\omega)$ does not necessarily provide information about the physical dimensionality of light. For instance, the full polarization matrix of a circularly polarized light beam involves just a single nonzero eigenvalue, whereas its real part satisfies $a_1 = a_2$ and $a_3 = 0$, thereby corresponding to (isotropic) 2D light in view of Eq. (5). Likewise, the complex polarization matrix of an incoherent and orthogonal superposition of a circularly polarized and a linearly polarized beam has two nonnegative eigenvalues, while in this case all three eigenvalues of the real-valued polarization matrix are nonzero. Hence, according to Eq. (6), the superposed field is genuinely 3D in character.

Polarimetric dimension

Although Eqs. (4)–(6) establish the definitions for the dimensionality of a light field, they do not provide information how 1D-, 2D-, or 3D-like the light in question is. For example, an elliptically polarized beam is formally two dimensional, but from a practical point of view it can be regarded as one dimensional if the polarization ellipse is highly squeezed (cf. linear polarization). Therefore, to characterize the dimensional nature of a light field more quantitatively, we introduce the spectral polarimetric dimension, $D(\mathbf{r},\omega)$, via the relation

$$D(\mathbf{r},\omega) = 3 - 2d(\mathbf{r},\omega), \tag{7}$$

where $d(\mathbf{r},\omega)$ is the distance between the real-valued matrix $\boldsymbol{\Phi}'(\mathbf{r},\omega)$ and the identity matrix associated with isotropic 3D light, i.e.,

$$d(\mathbf{r},\omega) = \sqrt{\frac{3}{2}\left[\frac{\mathrm{tr}\,\boldsymbol{\Phi}'^2(\mathbf{r},\omega)}{\mathrm{tr}^2\boldsymbol{\Phi}'(\mathbf{r},\omega)} - \frac{1}{3}\right]}, \tag{8}$$

with the scaling chosen so that $0 \leq d(\mathbf{r},\omega) \leq 1$. We remark that an expression formally similar to Eq. (8), with $\boldsymbol{\Phi}(\mathbf{r},\omega)$ replacing $\boldsymbol{\Phi}'(\mathbf{r},\omega)$, has been employed to characterize the degree of polarization of random 3D light fields [20, 21]. The polarimetric dimension is thus a real number that obeys $1 \leq D(\mathbf{r},\omega) \leq 3$. Moreover, it is invariant under orthogonal transformations, but generally not under unitary operations, since the latter may alter the polarization state and, consequently, the dimensionality of the light.

The physical meaning of $D(\mathbf{r},\omega)$ becomes more apparent by writing Eq. (7) in terms of the eigenvalues of $\mathbf{\Phi}'(\mathbf{r},\omega)$, viz.,

$$D(\mathbf{r},\omega)=3-\frac{\sqrt{2\left[(a_1-a_2)^2+(a_1-a_3)^2+(a_2-a_3)^2\right]}}{a_1+a_2+a_3}. \tag{9}$$

The above expression indicates that the minimum $D(\mathbf{r},\omega) = 1$ is always, and solely, encountered for 1D light ($a_2 = a_3 = 0$), while the maximum $D(\mathbf{r},\omega) = 3$ is reached if, and only if, the field is completely 3D isotropic ($a_1 = a_2 = a_3$). For 2D light ($a_3 = 0, a_2 > 0$), we find that $1 < D(\mathbf{r},\omega) \leq 2$, with the upper limit taking place when the two principal intensities are equal ($a_1 = a_2$). Values in the range $D(\mathbf{r},\omega) > 2$ are thereby clear signatures of 3D light [note that 3D light may nonetheless assume any value within the interval $1 < D(\mathbf{r},\omega) \leq 3$].

Since $D(\mathbf{r},\omega)$ is generally not an integer, it should not be identified as such with the actual physical dimensionality of the light [specified by Eqs. (4)–(6)], but as an effective dimension characterizing the intensity-distribution spread. Figure 1 provides an interpretative illustration for the polarimetric dimension, in which principal-intensity distributions for three different 3D light fields have been depicted. In the left panel a_1 is significantly larger than the intensities in the other directions, whereupon the light is effectively one dimensional and thus $D(\mathbf{r},\omega) \approx 1$. A practical realization of such a field would be a directional surface plasmon polariton beam [22–24]. In the middle panel $a_1 \approx a_2 \gg a_3$, indicating that the light field is virtually 2D isotropic and hence $D(\mathbf{r},\omega) \approx 2$. An unpolarized or a circularly polarized light beam of high degree of directionality [17, 25] would constitute an example. In the right panel all principal intensities are about equally distributed, which corresponds to isotropic 3D light and thereby yields $D(\mathbf{r},\omega) \approx 3$, as is the case for instance with blackbody radiation.

Examples

As concrete examples, we investigate the dimensionality of stationary polarized light and an evanescent wave created in total internal reflection.

Polarized random light

Let $E_\alpha(\mathbf{r},\omega)$ with $\alpha \in \{x,y,z\}$ represent a Cartesian component of the electric-field realization. Furthermore, let

$$\mu_{\alpha\beta}(\mathbf{r},\omega) = |\mu_{\alpha\beta}(\mathbf{r},\omega)|e^{i\varphi_{\alpha\beta}(\mathbf{r},\omega)} = \frac{\left\langle E_\alpha^*(\mathbf{r},\omega)E_\beta(\mathbf{r},\omega)\right\rangle}{\sqrt{\left\langle|E_\alpha(\mathbf{r},\omega)|^2\right\rangle\left\langle|E_\beta(\mathbf{r},\omega)|^2\right\rangle}}, \quad \alpha,\beta \in \{x,y,z\}, \tag{10}$$

where $\varphi_{\alpha\beta}(\mathbf{r},\omega)$ are real-valued phase factors, be the complex correlation coefficient between the α and β components. Since for polarized light all field components are completely correlated [20], i.e., $|\mu_{\alpha\beta}(\mathbf{r},\omega)| = 1$, we can express the polarization matrix in Eq. (1) of a polarized light field as

$$\mathbf{\Phi}(\mathbf{r},\omega) = \begin{pmatrix} I_x & \sqrt{I_xI_y}e^{i\varphi_{xy}} & \sqrt{I_xI_z}e^{i\varphi_{xz}} \\ \sqrt{I_xI_y}e^{-i\varphi_{xy}} & I_y & \sqrt{I_yI_z}e^{i\varphi_{yz}} \\ \sqrt{I_xI_z}e^{-i\varphi_{xz}} & \sqrt{I_yI_z}e^{-i\varphi_{yz}} & I_z \end{pmatrix}, \tag{11}$$

in which the shorthand notations $I_\alpha = \langle|E_\alpha(\mathbf{r},\omega)|^2\rangle$ and $\varphi_{\alpha\beta} = \varphi_{\alpha\beta}(\mathbf{r},\omega)$ have been introduced for convenience, and the phases satisfy [26]

$$\varphi_{xy} - \varphi_{xz} + \varphi_{yz} = 2m\pi, \quad m \in \mathbb{Z}. \tag{12}$$

By taking the real part of Eq. (11) and utilizing Eq. (12) one then gets that

$$\det\mathbf{\Phi}'(\mathbf{r},\omega) = I_xI_yI_z\left[1-\left(c_{xy}^2+c_{xz}^2+c_{yz}^2\right)+2c_{xy}c_{xz}c_{yz}\right]=0, \tag{13}$$

where $c_{\alpha\beta} = \cos\varphi_{\alpha\beta}$, implying that polarized light is necessarily 1D or 2D in nature. In other words, fully polarized 3D light does not exist.

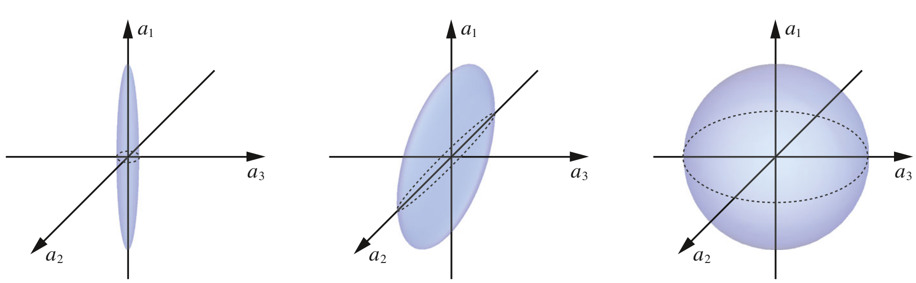

Fig. 1 (Color online) Examples of principal-intensity distributions for 3D light fields with $D(\mathbf{r},\omega) \approx 1$ (left panel), $D(\mathbf{r},\omega) \approx 2$ (middle panel), and $D(\mathbf{r},\omega) \approx 3$ (right panel)

This finding can intuitively be justified as follows. If a random light field with three electric components is spectrally fully polarized, it is at frequency ω represented by an ensemble of monochromatic field realizations as

$$\left\{\mathbf{E}(\mathbf{r},\omega)e^{-i\omega t}\right\} = \{E(\mathbf{r},\omega)\}\mathbf{e}(\mathbf{r},\omega)e^{-i\omega t}, \tag{14}$$

where $E(\mathbf{r},\omega)$ is a random scalar variable and $\mathbf{e}(\mathbf{r},\omega)$ is a deterministic three-component vector. The realizations thus have identical polarization states although their spectral densities may vary, and because the electric-field vector of monochromatic light is necessarily bounded in a plane [27], each realization lies in the same plane. Consequently, the random light that these monochromatic fields represent must fluctuate in this plane as well.

Random evanescent wave

As a second example, we consider an optical evanescent wave excited in total internal reflection at a planar dielectric interface ($z = 0$) by a stationary beam [28]. Both medium 1 ($z > 0$) and medium 2 ($z < 0$), having refractive indices $n_1(\omega)$ and $n_2(\omega)$, respectively, are taken lossless, and the x axis is chosen to coincide with the surface-propagation direction. Moreover, the incoming beam, generally carrying both an s-polarized and a p-polarized constituent, hits the boundary at the angle of incidence $\theta(\omega)$ that satisfies $\theta_c(\omega) < \theta(\omega) < \pi/2$, with $\theta_c(\omega) = \arcsin\tilde{n}^{-1}(\omega)$ being the critical angle and $\tilde{n}(\omega) = n_1(\omega)/n_2(\omega) > 1$. Under these conditions, the spatial part of the electric-field realization for the evanescent wave takes on in Cartesian coordinates the form [4, 5]

$$\mathbf{E}(\mathbf{r},\omega) = \frac{1}{\chi(\omega)}\begin{bmatrix} -i\gamma(\omega)t_p(\omega)E_p(\omega) \\ \chi(\omega)t_s(\omega)E_s(\omega) \\ \sin\theta(\omega)t_p(\omega)E_p(\omega)\end{bmatrix} e^{ik_1(\omega)\sin\theta(\omega)x}e^{-k_1(\omega)\gamma(\omega)z}, \tag{15}$$

where $E_s(\omega)$ and $E_p(\omega)$ are, respectively, the complex field amplitudes of the s- and p-polarized components of the incident light. Furthermore,

$$\begin{aligned}\chi(\omega) &= \sqrt{\sin^2\theta(\omega) + \gamma^2(\omega)},\\ \gamma(\omega) &= \tilde{n}^{-1}(\omega)\sqrt{\tilde{n}^2(\omega)\sin^2\theta(\omega) - 1},\end{aligned} \tag{16}$$

with $\gamma(\omega)$ being the decay constant of the evanescent wave, and

$$\begin{aligned}t_s(\omega) &= \frac{2\cos\theta(\omega)}{\cos\theta(\omega) + i\gamma(\omega)},\\ t_p(\omega) &= \frac{2\tilde{n}^2(\omega)\cos\theta(\omega)\chi(\omega)}{\cos\theta(\omega) + i\tilde{n}^2(\omega)\gamma(\omega)}\end{aligned} \tag{17}$$

are the Fresnel transmission coefficients of the two polarizations, and $k_1(\omega)$ is the wave number in medium 1.

On next calculating the polarization matrix in Eq. (1) for the evanescent wave given by Eq. (15), and then extracting the real part $\mathbf{\Phi}'(\mathbf{r},\omega) = \mathbf{\Phi}'(z,\omega)$ from the obtained expression, we find that

$$\det\mathbf{\Phi}'(z,\omega) = \frac{\sin^2\theta(\omega)\gamma^2(\omega)}{\chi^4(\omega)}w_s(z,\omega)w_p^2(z,\omega)\left[1 - |\mu(\omega)|^2\right]. \tag{18}$$

Above, $w_\nu(z,\omega) = |t_\nu(\omega)|^2 I_\nu(\omega)e^{-2k_1(\omega)\gamma(\omega)z}$ is proportional to the energy density of the $\nu \in \{s,p\}$ polarized part of the evanescent wave at height z, with $I_\nu(\omega) = \langle|E_\nu(\omega)|^2\rangle$ being the intensity of the respective component of the incoming beam, and $\mu(\omega) = \langle E_s^*(\omega)E_p(\omega)\rangle/\sqrt{I_s(\omega)I_p(\omega)}$ is the correlation coefficient among the s- and p-polarized constituents of the incident light. Equation (18) especially shows that $\det\mathbf{\Phi}'(z,\omega) = 0$ only when the excitation beam is totally polarized, i.e., $I_s(\omega) = 0$, $I_p(\omega) = 0$, or $|\mu(\omega)| = 1$, and in this case the ensuing evanescent wave is either 1D or 2D in character. Generally, however, when the incident beam is partially polarized [$I_s(\omega) \neq 0$, $I_p(\omega) \neq 0$, and $|\mu(\omega)| \neq 1$], we obtain from Eq. (18) that $\det\mathbf{\Phi}'(z,\omega) > 0$, corresponding to genuine 3D light. This discovery reveals that optical evanescent waves are predominantly 3D light fields, which necessitate a rigorous 3D treatment to fully describe their electromagnetic properties.

Motivated by the above result we further examine how close to isotropic 3D light an evanescent wave can be, and to this end we employ the polarimetric dimension $D(\mathbf{r},\omega)$ defined in Eq. (7). Utilizing Eqs. (15)–(17) then yields that the fundamental upper limit that $D(\mathbf{r},\omega)$ can attain for such a wave is

$$D(\mathbf{r},\omega) = 3 - \frac{2}{\sqrt{1 + 3\tilde{n}^4(\omega)\chi^4(\omega)}}, \tag{19}$$

which is reached when the incident light possesses the properties

$$|\mu(\omega)| = 0, \quad \frac{I_s(\omega)}{I_p(\omega)} = \left[\frac{\sin^4\theta(\omega) + \gamma^4(\omega)}{\chi^4(\omega)}\right]\left[\frac{|t_p(\omega)|}{|t_s(\omega)|}\right]^2. \tag{20}$$

For a high refractive-index contrast surface, such as GaP and air with $\tilde{n}(\omega) \approx 4$ in the optical regime [29], Eq. (19) shows that the polarimetric dimension may be as high as $D(\mathbf{r},\omega) \approx 2.96$, while for a typical SiO_2–air interface the maximum is around $D(\mathbf{r},\omega) \approx 2.67$.

Conclusions

In summary, we have formulated a framework to classify and to characterize the dimensionality of random light fields. To this end, it was shown that the number (1, 2, or 3) of nonzero eigenvalues of the real-valued polarization matrix $\mathbf{\Phi}'(\mathbf{r},\omega)$ [i.e. the real part of the full complex polarization matrix $\mathbf{\Phi}(\mathbf{r},\omega)$] specifies whether the light is 1D, 2D, or 3D, respectively. We also put forward a measure, the spectral polarimetric dimension, which quantifies the intensity-distribution spread and in this sense the effective dimensionality of a light field. The formalism was exemplified by showing that completely polarized random light is necessarily 1D or 2D in character, while an evanescent wave generated by a partially polarized beam in total internal reflection is unambiguously a genuine 3D light field. The polarimetric dimension could similarly be defined also in the space–time domain. Our work, providing novel insights and means to address polarization of random light fields, could thus be instrumental for applications involving complex-structured light, such as near-field optics and high-numerical-aperture imaging systems.

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Authors' contributions
The original ideas and results emerged from discussions among all the authors. AN and TS performed the calculations and wrote the manuscript with contributions from ATF and JJG. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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References
1. Brosseau, C: Fundamentals of Polarized Light: A Statistical Optics Approach. Wiley, New York (1998)
2. Gil, JJ, Ossikovski, R: Polarized Light and the Mueller Matrix Approach. CRC Press, Boca Raton (2016)
3. Setälä, T, Kaivola, M, Friberg, AT: Degree of polarization in near fields of thermal sources: effects of surface waves. Phys. Rev. Lett. **88**, 123902 (2002)
4. Norrman, A, Setälä, T, Friberg, AT: Partial spatial coherence and partial polarization in random evanescent fields on lossless interfaces. J. Opt. Soc. Am. A. **28**, 391–400 (2011)
5. Norrman, A, Setälä, T, Friberg, AT: Generation and electromagnetic coherence of unpolarized three-component light fields. Opt. Lett. **40**, 5216–5219 (2015)
6. Lindfors, K, Setälä, T, Kaivola, M, Friberg, AT: Degree of polarization in tightly focused optical fields. J. Opt. Soc. Am. A. **22**, 561–568 (2005)
7. Lindfors, K, Priimagi, A, Setälä, T, Shevchenko, A, Friberg, AT, Kaivola, M: Local polarization of tightly focused unpolarized light. Nat. Photon. **1**, 228–231 (2007)
8. Bauer, T, Banzer, P, Karimi, E, Orlov, S, Rubano, A, Marrucci, L, Santamato, E, Boyd, RW, Leuchs, G: Observation of optical polarization Möbius strips. Science. **347**, 964–966 (2015)
9. Bauer, T, Neugebauer, M, Leuchs, G, Banzer, P: Optical polarization Möbius strips and points of purely transverse spin density. Phys. Rev. Lett. **117**, 013601 (2016)
10. Leppänen, L-P, Friberg, AT, Setälä, T: Partial polarization of optical beams and near fields probed with a nanoscatterer. J. Opt. Soc. Am. A. **31**, 1627–1635 (2014)
11. Novotny, L, Beversluis, MR, Youngworth, KS, Brown, TG: Longitudinal field modes probed by single molecules. Phys. Rev. Lett. **86**, 5251–5254 (2001)
12. Dholakia, K, Čižmár, T: Shaping the future of manipulation. Nat. Photon. **5**, 335–342 (2011)
13. Tervo, J, Setälä, T, Friberg, AT: Theory of partially coherent electromagnetic fields in the space–frequency domain. J. Opt. Soc. Am. A. **21**, 2205–2215 (2004)
14. Voipio, T, Setälä, T, Friberg, AT: Partial polarization theory of pulsed optical beams. J. Opt. Soc. Am. A. **30**, 71–81 (2013)
15. Mandel, L, Wolf, E: Optical Coherence and Quantum Optics. Cambridge University Press, Cambridge (1995)
16. Dennis, MR: Geometric interpretation of the three-dimensional coherence matrix for nonparaxial polarization. J. Opt. A: Pure Appl. Opt. **6**, S26–S31 (2004)
17. Gil, JJ: Interpretation of the coherency matrix for three-dimensional polarization states. Phys. Rev. A. **90**, 043858 (2014)
18. Gil, JJ: Intrinsic Stokes parameters for 3D and 2D polarization states. J. Eur. Opt. Soc.-Rapid. **10**, 15054 (2015)
19. Horn, RA, Johnson, CR: Matrix Analysis. 2nd edition. Cambridge University Press, Cambridge (1985)
20. Setälä, T, Schevchenko, A, Kaivola, M, Friberg, AT: Degree of polarization for optical near fields. Phys. Rev. E. **66**, 016615 (2002)
21. Luis, A: Degree of polarization for three-dimensional fields as a distance between correlation matrices. Opt. Comm. **253**, 10–14 (2005)
22. Maier, SA: Plasmonics: Fundamentals and Applications. Springer, Berlin (2007)
23. Martinez-Herrero, R, Garcia-Ruiz, A, Manjavacas, A: Parametric characterization of surface plasmon polaritons at a lossy interface. Opt. Express. **23**, 4–28583 (2015)
24. Martinez-Herrero, R, Manjavacas, A: Basis for paraxial surface-plasmon-polariton packets. Phys. Rev. A. **94**, 063829 (2016)
25. Gil, JJ: Components of purity of a three-dimensional polarization state. J. Opt. Soc. Am. A. **33**, 40–43 (2016)
26. Setälä, T, Tervo, J, Friberg, AT: Complete electromagnetic coherence in the space–frequency domain. Opt. Lett. **29**, 328–330 (2004)
27. Born, M, Wolf, E: Principles of Optics. 7th (expanded) edition, Sec. 1.4.3. Cambridge University Press, Cambridge (1999)
28. de Fornel, F: Evanescent Waves: From Newtonian Optics to Atomic Optics. Springer, Berlin (2001)
29. Palik, ED, (ed.): Handbook of Optical Constants of Solids. Academic Press, San Diego (1998)

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