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Sparse kronecker pascal measurement matrices for compressive imaging

Yilin Jiang*, Qi Tong, Haiyan Wang and Qingbo Ji

Abstract

Background: The construction of measurement matrix becomes a focus in compressed sensing (CS) theory. Although random matrices have been theoretically and practically shown to reconstruct signals, it is still necessary to study the more promising deterministic measurement matrix.

Methods: In this paper, a new method to construct a simple and efficient deterministic measurement matrix, sparse kronecker pascal (SKP) measurement matrix, is proposed, which is based on the kronecker product and the pascal matrix.

Results: Simulation results show that the reconstruction performance of the SKP measurement matrices is superior to that of the random Gaussian measurement matrices and random Bernoulli measurement matrices.

Conclusions: The SKP measurement matrix can be applied to reconstruct high-dimensional signals such as natural images. And the reconstruction performance of the SKP measurement matrix with a proper pascal matrix outperforms the random measurement matrices.

Keywords: Compressed sensing, Deterministic measurement matrix, Kronecker product, Pascal matrix

Background

Compressed sensing (CS) theory is a novel sampling scheme, which indicates that a sparse signal can be recovered from much fewer samples than conventional method [1, 2]. The sampling and the compression procedure are completed by the linear projection in CS. In matrix notation, it can be expressed as

$$y = \Phi x \tag{1}$$

where $x \in \mathbb{R}^N$ is the original signal, Φ is an $M \times N (M \ll N)$ measurement matrix, $y \in \mathbb{R}^M$ is the measurement vector. x is said to be K-sparse if $\|x\|_0 \leq K$. CS theory asserts that if the measurement matrix Φ satisfies some conditions, the signal x can be recovered from measurements y without distortion.

The emergence of CS provides a new inspiration for optical imaging. Actually most of the nature images are compressible in terms of some sparsity basis, such as Discrete cosine transform (DCT) and Discrete wavelet transform (DWT). The compressibility of the real-word images shows the potential for optical compressive imaging. In the past few years, CS technique has made great progress in many research fields, which include terahertz compressive imaging [3], spectral imaging [4], single pixel imaging [5] and infrared imaging [6]. Some optical imaging applications have been implemented in specific physical experiments.

Measurement matrix construction is a crucial problem in CS. The measurements obtained by measurement matrix are related to whether the signal can be accurately reconstructed. If there is enough information within the measurements, the signal can be recovered with high probability. Random measurement matrices are proved to have the merit of universality but suffer from several shortcomings. Firstly, random measurement matrices consume lots of storage resources. Secondly, there is no feasible algorithm to verify whether the random matrix satisfies the requirement as a measurement matrix [7, 8]. The research on deterministic sampling can be tracked back to the binary matrices via polynomials over finite field [9]. The deterministic measurement matrix has the superiority in physical implementation and the advantage of saving storage space. Therefore, many researches on

* Correspondence: jiangyilin@hrbeu.edu.cn
College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China

the deterministic measurement matrix construction have been carried out. Lu introduced a construction of ternary matrices with small coherence [10]. Yao presented a novel simple and efficient measurement matrix named incoherence rotated chaotic matrix [11]. Huang proposed a symmetric Toeplitz measurement matrix [12]. Zhao introduced a deterministic complex measurement matrix to sample the signals in the single pixel imaging [13].

In this paper, we propose a new construction method of deterministic measurement matrix, termed sparse kronecker pascal (SKP) measurement matrix. The SKP measurement matrix combines the properties of the kronecker product and the pascal matrix. It is suitable for the reconstruction of natural images, which are usually high-dimensional signals. Simulations and analyses confirm that the SKP measurement matrices can reconstruct the natural images with a better performance.

Methods

The SKP measurement matrix construction

In mathematics, the kronecker product is an operation on two matrices of arbitrary size resulting in a block matrix [14].

Definition: If A is an $m \times n$ matrix, B is a $p \times q$ matrix, then the kronecker product $\mathrm{A} \otimes \mathrm{B}$ is the $mp \times nq$ block matrix. It can be expressed as

$$\mathrm{A}\otimes\mathrm{B} = \begin{bmatrix} a_{11}\mathrm{B}\cdots a_{1n}\mathrm{B} \\ \vdots \quad \ddots \quad \vdots \\ a_{m1}\mathrm{B}\cdots a_{mn}\mathrm{B} \end{bmatrix} \tag{2}$$

Pascal matrix is a symmetric positive definite matrix with integer entries taken from pascal's triangle [15]. The 4×4 truncations of these are shown below

$$\mathrm{P}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix} \tag{3}$$

it can be seen clearly that the entries near the diagonal of the pascal matrix increase with a geometric growth. It is effective to achieve sparse purpose by the kronecker product. Based on the pascal matrix and the kronecker product, we present the SKP matrix

$$\mathrm{H} = k * \mathrm{I}\otimes\mathrm{P} = k \begin{bmatrix} \mathrm{P} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & \vdots \\ \vdots & 0 & \mathrm{P} & 0 & \vdots \\ \vdots & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \mathrm{P} \end{bmatrix} \tag{4}$$

where H is the proposed SKP matrix, k signifies the scaling factor, I represents an identity matrix, P denotes the pascal matrix. Suppose that I is a $q \times q$ matrix, P is a $p \times p$ matrix, then H is a $pq \times pq$ matrix. We can get an $m \times n$ SKP measurement matrix Φ by selecting appropriately m rows from H for CS, here $n = p \times q$, m < n.

Now we describe how to select right rows from H to construct various dimensional measurement matrices. The selection method is to follow the principle of equal interval, which can improve the irrelevance between the selected row vectors. From the first row, we can construct the measurement matrix of multiple dimensions by choosing different interval lengths. If the interval length $d = 2, p = 4, q = 64$, the size of the SKP measurement matrix is 128×256. Similarly, when the interval length $d = 3$, the size becomes 86×256.

In CS, the measurement matrix must satisfy certain conditions. Candes and Tao propose a criterion named restricted isometry property (RIP) [16, 17]. A measurement matrix is said to satisfy the RIP of order K if there exists a constant $\delta_K \in (0, 1)$ such that

$$(1-\delta_K)\|x\|_2^2 \le \|\Phi x\|_2^2 \le (1+\delta_K)\|x\|_2^2 \tag{5}$$

for any K-sparse vector x. It is similar to that any K column vectors of the measurement matrix Φ are linearly independent. The RIP criterion guarantees that the sparse signal can be recovered exactly from the measurements.

The SKP measurement matrix is a particular matrix. The determinant of every P_n is 1 and the determinant of SKP matrix H is k_1 ($k_1 \neq 0$), which signify that any column vectors or row vectors from $\mathrm{P_n}$ and H are linearly independent. Therefore the SKP measurement matrix Φ is also a linear independent system between row vectors. The correlation among the resulting measurements is reduced, and the unique distribution of the SKP measurement matrix facilitates its implementation.

Results and discussion

In this part, we conduct numerical experiments to validate the performance of the SKP measurement matrix. The test images are of size 256×256 pixels. Orthogonal matching pursuit (OMP) algorithm is chosen as the recovery algorithm [18]. The sparsity basis Ψ is selected as the DCT matrix. Reconstruction processes are implemented in MATLAB R2016a. The size of the pascal matrix is 4×4, the scaling factor $k = 0.05$ and the identity matrix I is 64×64. Firstly the interval length is set to $d = 2$. We compare the reconstruction performance among the SKP measurement matrix, the random Gaussian measurement matrix and random Bernoulli measurement matrix. The quality of reconstructed images is measured by the peak signal-to-noise ratio (PSNR) in Eq. (7)

$$MSE = \frac{1}{N}\sum |x - x_{recons}|^2 \tag{6}$$

Fig. 1 Reconstructed Lena images with different measurement matrices at compression ratio (m/n) 0.5: **a** Original image; **b** Gaussian measurement matrix (22.8972 dB); **c** Bernoulli measurement matrix (23.3919 dB); **d** SKP measurement matrix (27.1355 dB)

$$PSNR = 20\log\left(\frac{255}{\sqrt{MSE}}\right) \tag{7}$$

Simulation results are shown in Figs. 1 and 2. It can be observed that the reconstructed images using the SKP measurement matrix is the clearest among all the reconstructed images. The reconstructed images by the random Gaussian measurement matrices and random Bernoulli measurement matrices are blurry and lose some details compared to the SKP measurement matrix. In addition, the differences between the reconstructed images are also very obvious in terms of PSNR values. The PSNR values of reconstructed images by the SKP measurement matrix are almost 4 dB higher than that by the random measurement matrices. Figs. 1 and 2 demonstrate that the SKP measurement matrix outperforms the random Gaussian measurement matrices and random Bernoulli measurement matrices at the compression ratio of 0.5.

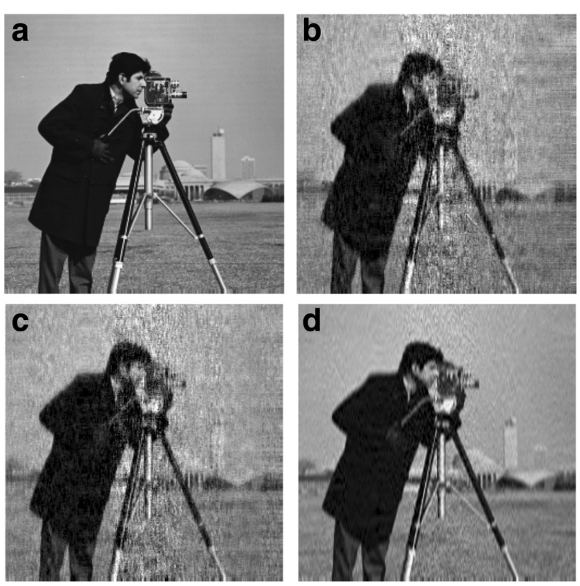

Fig. 2 Reconstructed Cameraman images with different measurement matrices at compression ratio (m/n) 0.5: **a** Original image; **b** Gaussian measurement matrix (20.1773 dB); **c** Bernoulli measurement matrix (20.0256 dB); **d** SKP measurement matrix (24.4332 dB)

Table 1 PSNR (in dB) values of reconstructed images under different experimental conditions

	compression ratio 0.33			compression ratio 0.5		
size	2	4	8	2	4	8
Lena						
Gaussian	20.2904	20.5879	20.2732	23.1280	23.1937	23.2459
Bernoulli	20.2536	20.6113	20.5530	22.8623	23.2264	23.1766
SKP	25.1923	23.8560	19.3820	28.3688	27.1355	21.9910
Cameraman						
Gaussian	17.7601	17.8477	17.8135	20.0125	20.3295	20.2115
Bernoulli	17.9861	17.9031	17.9366	20.0274	20.2517	20.6806
SKP	22.7272	21.9731	19.1054	25.5244	24.4332	19.6751

The further results present in Table 1. Table 1 shows more PSNR values of reconstructed images. And the measurement matrices include the random Gaussian measurement matrices, random Bernoulli measurement matrices and the SKP measurement matrices. In this part, the size of the pascal matrix is considered.

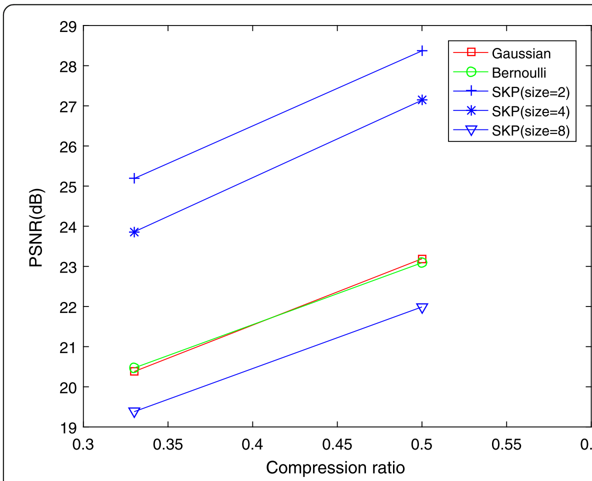

Fig. 3 The reconstruction accuracy of Lena image between each measurement matrix under different compression ratios

It can be seen from Table 1 when the size of the pascal matrix is 2 or 4, the reconstruction property of the SKP measurement matrices is better than that of the random Gaussian measurement matrices and random Bernoulli measurement matrices from PSNR values. When the size of the pascal matrix is 8, the reconstruction performance of the SKP measurement matrices has a serious decline or even less than the random Gaussian measurement matrices and random Bernoulli measurement matrices, which is caused by the further weakening of the orthogonality between row vectors of the SKP measurement matrices. Thus, the SKP measurement matrix construction needs to consider the influence of the pascal matrix dimension. The reconstruction accuracy of Lena image between each measurement matrix under different compression ratios is shown intuitively in Fig. 3.

Conclusions

In this paper, a new deterministic measurement matrix, SKP measurement matrix, is proposed for compressive imaging. The SKP measurement matrix has the advantages of simple structure, less storage space and convenient physical implementation, which offer great potential for compressive imaging applications. And we find that the size of the pascal matrix affects the reconstruction performance of the SKP measurement matrix. Simulation results demonstrate that the SKP measurement matrix with a proper pascal matrix can be used to effectively reconstruct the natural images and outperforms the random measurement matrices.

Abbreviations

CS: Compressed sensing; DCT: Discrete cosine transform; DWT: Discrete wavelet transform; OMP: Orthogonal matching pursuit; PSNR: Peak signal-to-noise ratio; RIP: Restricted isometry property; SKP: Sparse kronecker pascal

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article and its additional file.

Authors' contributions

All the authors make contribution to this work. YJ and QT conceived the idea and wrote the manuscript; HW designed the experiments and analyzed the data; QJ revised the manuscript. All authors read and approved the final manuscript.

Consent for publication

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Competing interests

The authors declare that they have no competing interests.

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