

RESEARCH

Open Access



# Absolute surface metrology by shear rotation with position error correction

Weibo Wang<sup>1,2\*</sup>, Biwei Wu<sup>1</sup>, Pengfei Liu<sup>1</sup>, Dong Huo<sup>1</sup> and Jiubin Tan<sup>1</sup>

## Abstract

**Background:** Absolute test is one of the most important and efficient techniques to separate the reference surface which usually limits the accuracy of test results.

**Method:** For the position error correction in absolute interferometry tests based on rotational and translational shears, the estimation algorithm adopts least-squares technique to eliminate azimuthal errors caused by rotation inaccuracy and the errors of angular orders are compensated with the help of Zernike polynomials fitting by an additional rotation measurement with a suitable selection of rotation angles.

**Results:** Experimental results show that the corrected results with azimuthal errors are very close to those with no errors, compared to the results before correction.

**Conclusions:** It can be seen clearly that the testing errors caused by rotation inaccuracy and alignment errors of the measurements can be consequently eliminated from the differences in measurement results by the proposed method.

**Keywords:** Absolute test, Shear rotation, Error correction, Zernike polynomials

## Background

In optical interferometric testing, the test surface map is not obtained independently but only in combination with the reference surface. Several ingenious techniques have been devised to obtain absolute surface measurements, e.g., two-sphere [1, 2] method for spherical reference surfaces and “three-flat” approach for flat surface [3]. However, the classic two-sphere method with cat’s-eye position measurement is sensitive to the lateral shear of the coma wavefront, which will introduce astigmatism and spherical terms [2]. For decades, the shift-rotation methods without the testing of cat’s-eye position have been developed to test spherical and flat surfaces [4–9]. These approaches yield an estimate for the test surface errors without changing experimental settings, such as cavity length, that may affect the apparent reference errors. The classic multi-angle averaging method proposed by Evans and Kestner, measures the spherical surface at  $N$  angular positions equally spaced with respect to the optical axis and the resulting wavefronts are averaged,

then errors in the rotated member with angular orders that are not integer multiples of the number of positions will be removed without Zernike fitting [10, 11].

It always assumes that there is no azimuthal position error during part rotation in the previous absolute test methods. However, the rotations of the test part introduce uncertainties related to azimuthal errors of the rotational angle and lateral displacement of the part with respect to the optical axis of the interferometer [11]. Moreover, rotation should be very precise when higher order spatial frequency terms are required, which are particularly sensitive to azimuthal position errors. In practice, there are challenges to rotate the test surface accurately to the desired positions, especially for large optics, and keep the environment and metrology system stable during the multi-measurements [12]. So we present a method to determine the true azimuthal positions of part rotation and consequently eliminate testing errors caused by rotation inaccuracy.

## Method

The shearing test is based on the analysis of differences in measurement results that occur when rotating or translating the test surface. The test results

\* Correspondence: weibo.wang@eng.ox.ac.uk

<sup>1</sup>Institute of Ultra-precision Optoelectronic Instrument Engineering, Harbin Institute of Technology, Harbin 150001, China

<sup>2</sup>Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK

yield a collection of error maps. Each error map describes the sum of apparent reference errors and test surface errors for a particular position and orientation of the test surface. If the test part is rotated to  $N$  equally spaced positions about the optical axis and the resulting, we can get the averaged wavefront

$$\begin{aligned} T_{ave}(\rho, \theta) &= \frac{1}{N} \sum_{i=0}^{N-1} T_i(\rho, \theta) \\ &= \frac{1}{N} \sum_{i=0}^{N-1} [R(\rho, \theta) + S(\rho, \theta)] \end{aligned} \quad (1)$$

where  $R(\rho, \theta)$  is the systematic error including the reference surface,  $S(\rho, \theta)$  is the surface error of the test part.

The wavefront of circular cross section can be expanded by polar coordinate polynomials in the following form

$$W(\rho, \theta) = \sum_{k,l} R_l^k(\rho) (\alpha_l^k \cos k\theta + \alpha_l^{-k} \sin k\theta) \quad (2)$$

where  $R_l^k(\rho)$  are the radial terms of Zernike polynomials and coefficients  $\alpha_l^{\pm k}$  specify the magnitude of each term while the angular terms specify the angular part of the polynomial representation.  $\rho$  and  $\theta$  are the normalized radial and angular coordinates.

From Eq. (2), if the wavefront is rotated to  $N$  equally spaced positions about the optical axis ( $\phi = 2\pi/N$ ), the averaged resulting wavefront can be written as

$$\begin{aligned} W_{ave}(\rho, \theta) &= \frac{1}{N} \sum_{j=0}^{N-1} W\left(\rho, \theta + j\frac{2\pi}{N}\right) \\ &= \frac{1}{N} \sum_{j,k,l} R_l^k(\rho) \left( \cos k\theta \sum_{j=0}^{N-1} \alpha_l^{j\phi,k} + \sin k\theta \sum_{j=0}^{N-1} \alpha_l^{j\phi,-k} \right) \end{aligned} \quad (3)$$

where

$$\begin{bmatrix} \alpha_l^{\phi,k} \\ \alpha_l^{\phi,-k} \end{bmatrix} = \begin{bmatrix} \cos k\phi & \sin k\phi \\ -\sin k\phi & \cos k\phi \end{bmatrix} \begin{bmatrix} \alpha_l^k \\ \alpha_l^{-k} \end{bmatrix} \quad (4)$$

For  $k=0$  (i.e., for rotationally symmetric terms), it is the intuitively obvious result that the procedure has no influence on rotationally symmetric terms. For  $k \neq 0$ , the series sum to zero for all  $\cos k\phi$  except  $k = cN$  ( $c=1,2,3,\dots$ ) and for all  $\sin k\phi$ . It is easy to see that rotating a wavefront to  $N$  equally spaced positions and averaging removes nonrotationally symmetric terms of all angular orders except  $kN\theta$ . The term  $W_{kN\theta}(\rho, \theta)$  is the  $N$ th rotationally symmetric component (angular orders  $kN\theta$ ), which can be written as

$$\begin{aligned} W_{kN\theta}(\rho, \theta) &= \sum_{k,l} (-1)^{k(N+1)} R_l^{kN}(\rho) (\alpha_l^{kN} \cos kN\theta \\ &\quad + \alpha_l^{-kN} \sin kN\theta) \end{aligned} \quad (5)$$

So the averaged test wavefront can be rewritten as

$$T_{ave}(\rho, \theta) = R(\rho, \theta) + S_{sym}(\rho, \theta) + W_{kN\theta}(\rho, \theta) \quad (6)$$

where  $S_{sym}(\rho, \theta)$  is the rotational symmetry surface deviation of the test part  $S(\rho, \theta)$ .

Furthermore, the asymmetric component of the test surface can be derived as

$$S_{asy}(\rho, \theta) = T_i(\rho, \theta) - T_{ave}(\rho, \theta) + W_{kN\theta}(\rho, \theta) \quad (7)$$

The errors of angular variation  $kN\theta$  can be represented based on Zernike polynomials and additional shear rotation measurement [9]. And it may be always neglected in the multi-angle averaging method, when  $N$  is large enough.

Additional measurements provide redundancies to improve and characterize measurement uncertainties. However, the rotation of the test part also introduces uncertainties related to azimuthal errors of the rotational angle and lateral displacement of the part with respect to the optical axis of the interferometer. The effect of uncertainties will arise from uncertainties in the rotational angle. Moreover, there are challenges to rotate the test surface accurately to the desired positions, especially for large optics, and keep the environment and metrology system stable during the multi-measurements.

So the estimation algorithm should be presentd to eliminate azimuthal errors caused by rotation inaccuracy. And the unknown relative alignment of the measurements also can be estimated through the differences in measurement results at overlapping areas.

The difference  $W$  between the shear rotation measurements can be written as

$$\begin{aligned} W &= R(\rho, \theta) + S_i(\rho, \theta) - R(\rho, \theta) - S_j(\rho, \theta + \phi) \\ &= S_i(\rho, \theta) - S_j(\rho, \theta + \phi) \\ &= \sum_{k,l} R_l^k(\rho) (\Delta\alpha_l^k \cos k\theta + \Delta\alpha_l^{-k} \sin k\theta) \end{aligned} \quad (8)$$

where  $\Delta\alpha_l^{\pm k}$  is the differences of the coefficients between two measurements.

It is trivially obvious to find  $\alpha_l^{\pm k}$  in terms of  $\Delta\alpha_l^{\pm k}$  from the difference of two measurements from Eqs. (4) and (8)

$$\alpha_l^{\pm k} = -\frac{1}{2} \left[ \Delta\alpha_l^{\pm k} \pm \frac{\Delta\alpha_l^{\mp k} \sin k\phi}{(1 - \cos k\phi)} \right] \quad (9)$$

This shows that the azimuthal terms of the wavefront can be determined from the azimuthal terms of the

difference between the original wavefront and itself after rotation by  $\phi$ . So the wavefront can be represented based on Zernike polynomials. Furthermore, the  $kN\theta$  variations of surface deviation  $W_{kN\theta}(\rho, \theta)$  neglected in the multi-angle averaging method can also be obtained by additional rotation testing with a suitable selection of rotation angles  $\theta_0$  with  $k = cN$  and  $k\theta_0 \neq 2m\pi$  ( $m$  is an integer).

The differences of the coefficients between two measurements can be written as

$$\Delta\alpha_l^{\pm k} = \alpha_l^{\pm k} (\cos k\phi_i - 1) \pm \alpha_l^{\mp k} \sin k\phi_i \tag{10}$$

For azimuthal position error correction, the angle  $\phi_i$  can be treated as additional unknowns together with the coefficients  $\alpha_l^{\pm k}$ . Then their actual values can be determined from the measured difference wavefront by least-squares method. Then the estimation algorithm adopts least-squares technique to eliminate azimuthal errors caused by rotation inaccuracy.

From Eq. (8), the wavefront difference can be further written as

$$\begin{aligned} W_i^k &= \sum_{k,l} R_l^k(\rho) \{ (\cos k\phi_i - 1) (\alpha_l^k \cos k\theta + \alpha_l^{-k} \sin k\theta) \\ &\quad + \sin k\phi_i (\alpha_l^{-k} \cos k\theta + \alpha_l^k \sin k\theta) \} \\ &= \sum_{k,l} \{ \gamma_{0l}^k Z_l^k(\rho, \theta) (\cos k\phi_i - 1) \\ &\quad + \tilde{\gamma}_{0l}^k Z_l^k(\rho, \theta) \sin k\phi_i \} = \sum_{k,l} \xi_{li}^k Z_l^k(\rho, \theta) \end{aligned} \tag{11}$$

The cost functions can be obtained by least squares method and be minimized to determine the true values of the unknowns of  $\gamma_{0l}^k$ ,  $\tilde{\gamma}_{0l}^k$  and  $\phi_i$ , as discussed in [12].

$$\begin{aligned} &\begin{bmatrix} \sum_{i=0}^{N-1} [\cos(k\phi_i) - 1]^2 & \sum_{i=0}^{N-1} \sin(k\phi_i) [\cos(k\phi_i) - 1] \\ \sum_{i=0}^{N-1} \sin(k\phi_i) [\cos(k\phi_i) - 1] & \sum_{i=0}^{N-1} \sin^2(k\phi_i) \end{bmatrix} \begin{bmatrix} \gamma_{0l}^k \\ \tilde{\gamma}_{0l}^k \end{bmatrix} \\ &= \begin{bmatrix} \sum_{i=0}^{N-1} \hat{X}_{li}^k [\cos(k\phi_i) - 1] & \sum_{i=0}^{N-1} \hat{X}_{li}^k \sin(k\phi_i) \end{bmatrix} \end{aligned} \tag{12}$$

$$\begin{bmatrix} \sum_l^{L(k)} [\gamma_{0l}^k]^2 & \sum_l^{L(k)} \gamma_{0l}^k \tilde{\gamma}_{0l}^k \\ \sum_l^{L(k)} \gamma_{0l}^k \tilde{\gamma}_{0l}^k & \sum_l^{L(k)} [\tilde{\gamma}_{0l}^k]^2 \end{bmatrix} \begin{bmatrix} \cos(k\phi_i) \\ \sin(k\phi_i) \end{bmatrix} = \begin{bmatrix} \sum_l^{L(k)} \{ \hat{X}_{li}^k \gamma_{0l}^k + [\gamma_{0l}^k]^2 \} \\ \sum_l^{L(k)} \{ \hat{X}_{li}^k \tilde{\gamma}_{0l}^k + \gamma_{0l}^k \tilde{\gamma}_{0l}^k \} \end{bmatrix} \tag{13}$$

This generalized algorithm adopts least-squares technique to determine the true azimuthal positions of part rotation and consequently eliminates testing

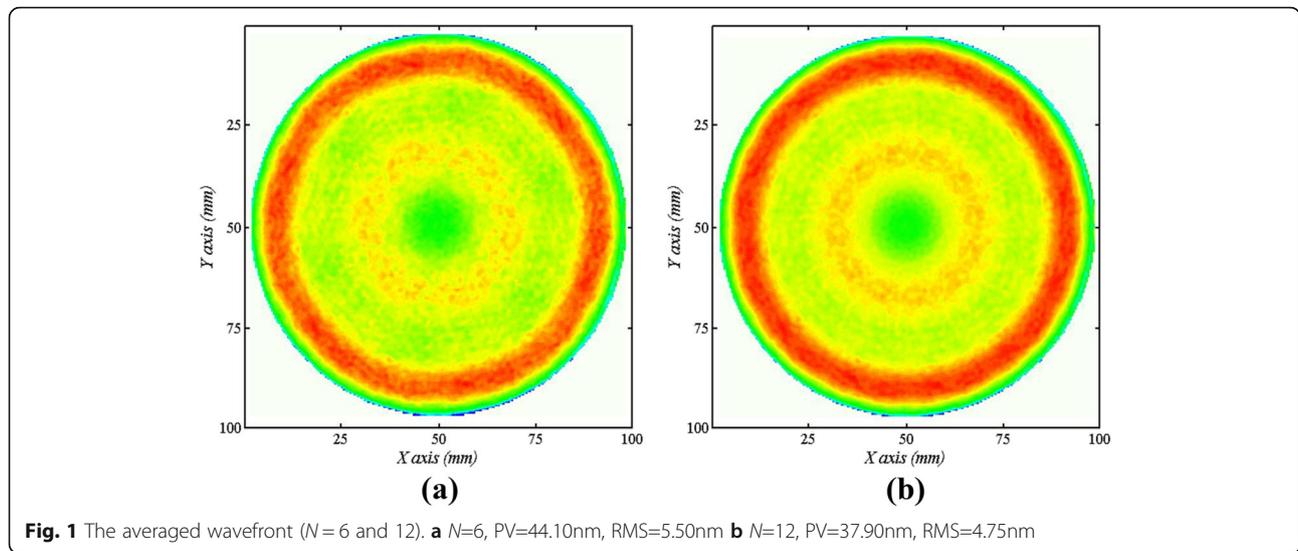
errors caused by rotation inaccuracy. The true values of the unknowns of  $\gamma_{0l}^k$ ,  $\tilde{\gamma}_{0l}^k$  and  $\phi_i$  can be obtained by the iterative procedure. The total computational time is influenced by the number of terms of Zernike polynomials in consideration (maximum  $l$  and  $k$ ), the number of rotation  $N$ , and the precision of the initial guess of  $\phi_i$ . Finally, the testing errors caused by rotation inaccuracy can be compensated by the solutions of  $\gamma_{0l}^k$ ,  $\tilde{\gamma}_{0l}^k$  and  $\phi_i$ .

### Results

For the verification of the described method, experiments are presented in a standard Fizeau interferometer. The surface under test is a spherical mirror with a clear aperture of 100 mm and surface error within  $\lambda/10PV$ . The accuracy of rotations can be better than  $0.1^\circ$  and the 5-Axis Mount of ZYGO Company can provide 13 mm X and Y adjustment, 50 mm Z adjustment and  $\pm 2^\circ$  tip and tilt adjustment. The spherical surface is tested at the normal testing position and various orientations with the classic multi-angle averaging method. These approaches can yield an estimate for the test surface errors without changing experimental settings, such as cavity length, that may affect the apparent reference errors.

The averaged wavefronts for  $N=6$  and 12 are shown in Fig. 1. The errors of angular orders  $kN\theta$  resembling a hexagon can be seen obviously from Fig. 1a, which may introduce unnecessary measurement errors when it is neglected in the absolute surface metrology. When  $N$  is large enough, the terms  $2n\pi/\phi$  are close to rotationally symmetric deviations, as shown in Fig. 1b. The errors of angular orders  $kN\theta$  can be quite small.

The differences of Fig. 1a and b are shown in Fig. 2a. It can see clearly that the averaged wavefront are suffering from the errors of angular orders  $kN\theta$ . For the compensation of  $W_{kN\theta}(\rho, \theta)$ , the additional rotation testing with a suitable selection of rotation angles is implemented. The  $W_{kN\theta}(\rho, \theta)$  of the test surface are restructured and compensated with the help of least-squares fitting of Zernike polynomials. The differences of Fig. 1a and b after  $W_{kN\theta}(\rho, \theta)$  compensation can be seen from Fig. 2b, and the compensated errors of angular variation  $kN\theta$  are shown in Fig. 3. The differences of Fig. 1a and b after compensation are very small. The errors of angular variation  $kN\theta$  have been well compensated. It implies that the described method with  $W_{kN\theta}(\rho, \theta)$  compensation can obtain high accuracy even with fewer rotation measurements. However, because of position errors, the errors caused by rotation inaccuracy still can be seen from Fig. 2b.



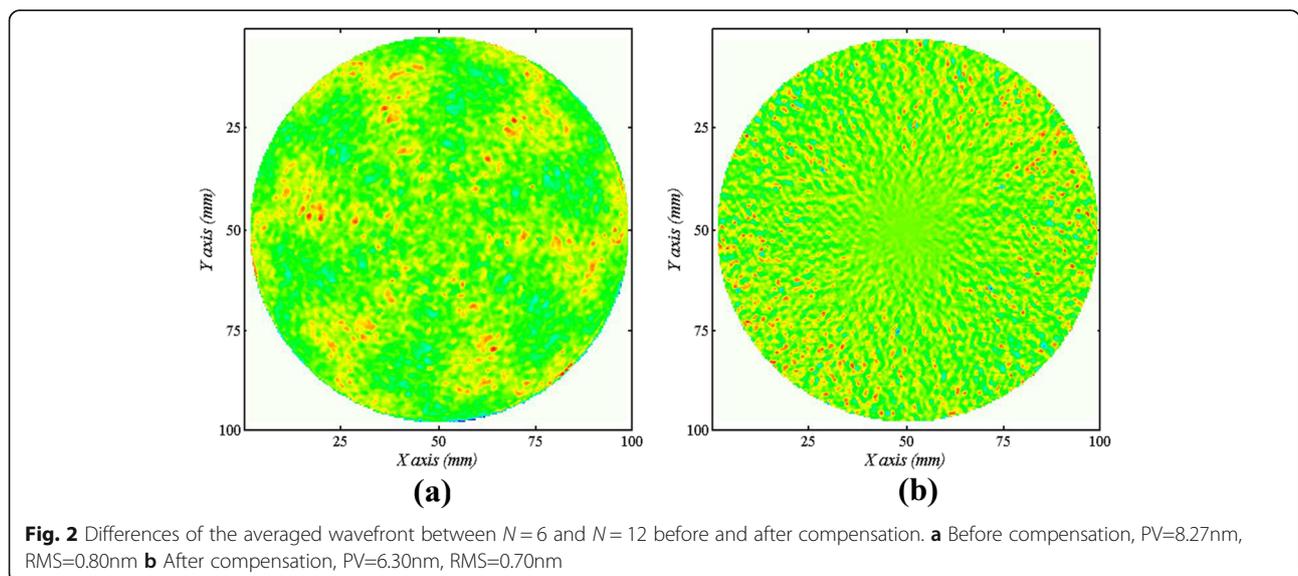
Furthermore, the averaged wavefront for  $N = 6$  with position errors (azimuthal errors and alignment error) introduced is shown in Fig. 4 and the difference of the averaged wavefront for  $N = 6$  before and after position errors introduced is shown in Fig. 5. Figures 1a and 4 have a similar distribution on optical path difference and some differences on PV and RMS. More details can be seen from Fig. 5. The test results are suffering from the position errors. As mentioned above, it's difficult to rotate the test surface accurately to the desired positions, especially for large optics. There are also many challenges to keep the environment and metrology system stable during the multi-averaging

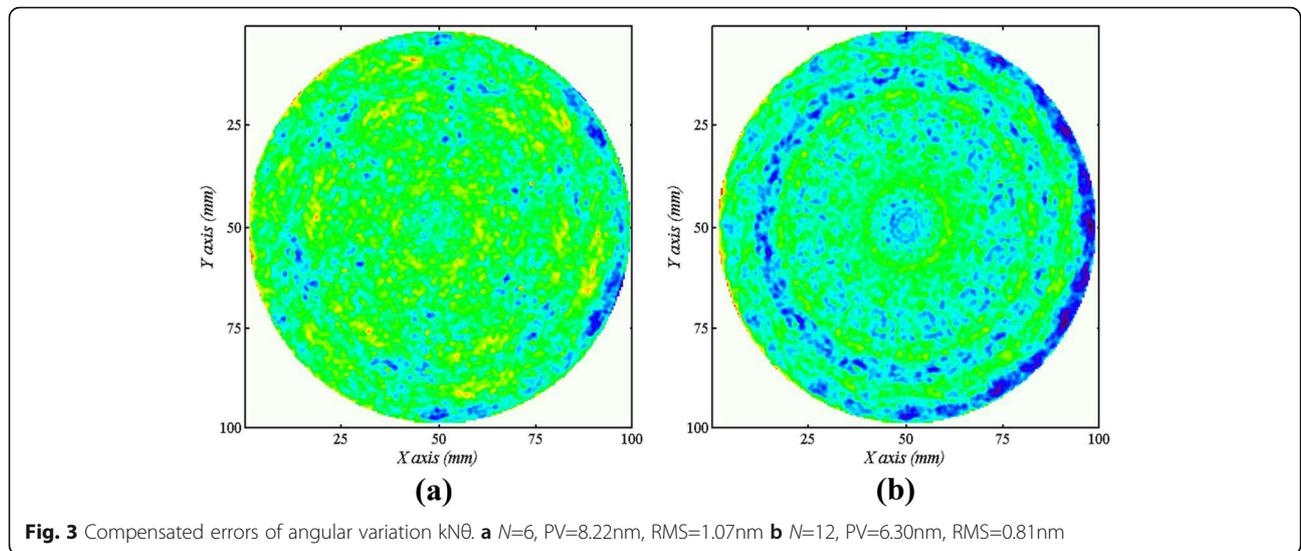
measurements, especially for large  $N$ . So the position error correction is necessary.

### Discussion

In order to correct the errors due to the rotation inaccuracy, the estimation algorithm adopts least-squares technique to determine the true azimuthal positions of part rotation and consequently eliminates testing errors caused by rotation inaccuracy. The surface is tested on the precision rotation stage with accurate position and random azimuthal errors within  $\pm 2^\circ$  respectively.

The Zernike coefficients of the results in absolute surface metrology are shown in Fig. 6. The Zernike coefficients with



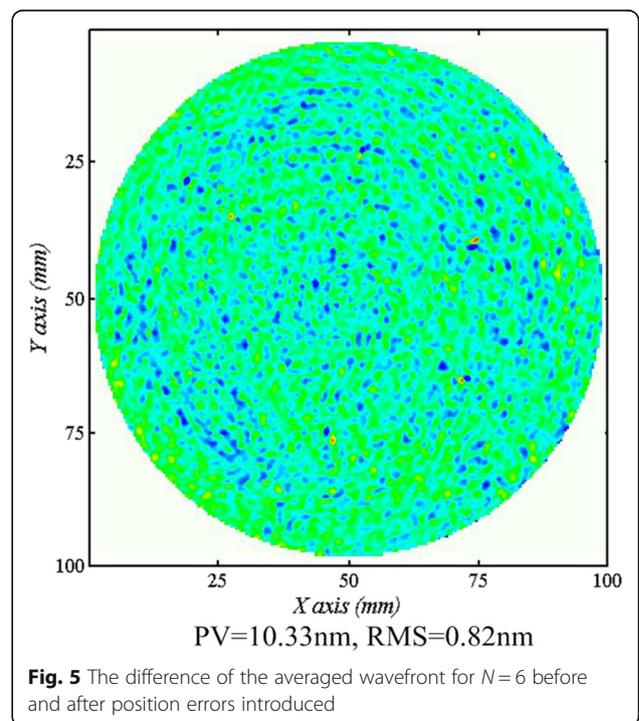
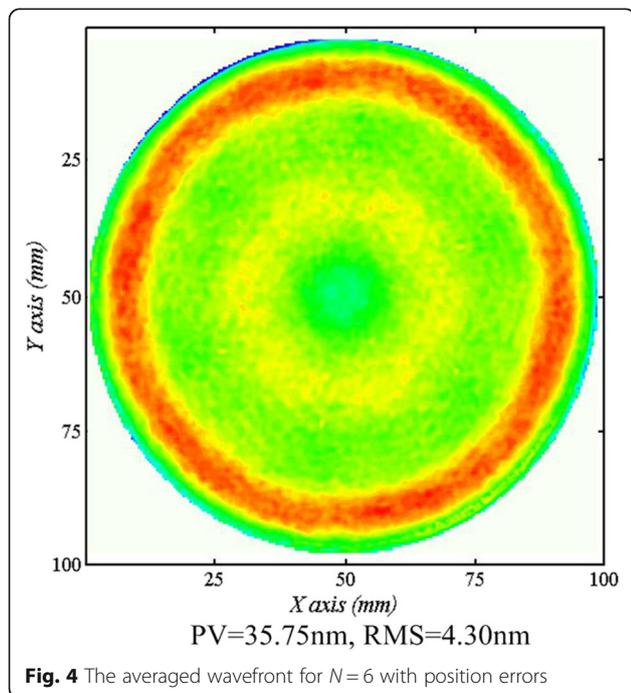


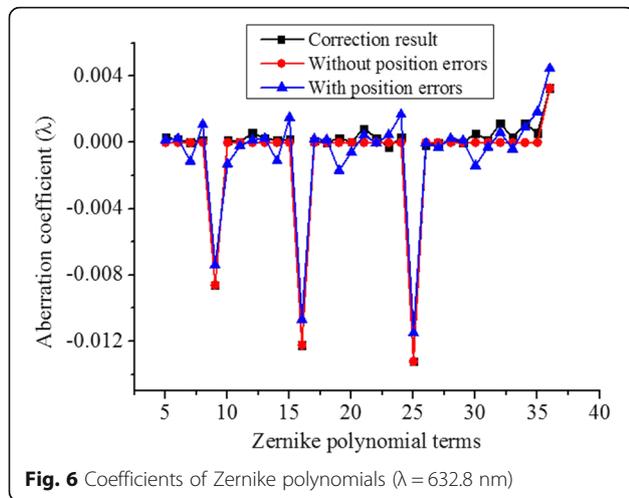
the correction of azimuthal errors and alignment errors are also shown in Fig. 6. The corrected Zernike coefficients are very close to those with fine adjustment and no additional azimuthal errors, compared to the results before correction. The coma terms ( $Z7 \sim Z8$ ,  $Z14 \sim Z15$ ,  $Z23 \sim Z24$ ) and spherical terms ( $Z9$ ,  $Z16$ ,  $Z25$ ,  $Z36$ ) introduced by the azimuthal errors and alignment errors have been well suppressed. It implies that the testing errors caused by rotation inaccuracy and alignment errors of the measurements can

be consequently eliminated from the differences in measurement results by the proposed method.

### Conclusions

We discussed the position error estimation algorithm to determine the true azimuthal positions of part rotation and the  $kN\theta$  compensation method to offer possibility to obtain high accuracy even with fewer rotation





**Fig. 6** Coefficients of Zernike polynomials ( $\lambda = 632.8$  nm)

measurements. It can be used to overcome the challenges of rotating the test surface accurately to the desired positions, especially for large optics and obtain the higher order spatial frequency terms required. Experimental results have been given to verify the effectiveness of the proposed method.

#### Funding

National Natural Science Foundation of China (51205089, 51275121 and 51475111), China Postdoctoral science foundation (2012 M520726), National Key Scientific Instrument and Equipment Development Project (2011YQ040087), China Scholarship Council (201406125121).

#### Authors' contributions

All authors have participated in the method discussion and result analysis. The experiments are conducted by JT. All authors have read and agreed with the contents of the final manuscript.

#### Competing interests

The authors declare that they have no competing interests.

Received: 29 September 2016 Accepted: 20 December 2016

Published online: 19 January 2017

#### References

- Jensen, A.E.: Absolute calibration method for Twyman-Green wavefront testing interferometers. *J. Opt. Soc. Am.* **63**, 1313A (1973)
- Selberg, L.A.: Absolute testing of spherical surfaces. In: *Optical Fabrication and Testing*, Vol. 13 of OSA 1994 Technical Digest Series, pp. 181–184. Optical Society of America, Washington, D.C (1994)
- Fritz, B.S.: Absolute calibration of an optical flat. *Opt. Eng.* **23**, 379–383 (1984)
- Freischlad, K.R.: Absolute interferometric testing based on reconstruction of rotational shear. *Appl. Opt.* **40**(10), 1637–1648 (2001)
- Bloemhof, E.E.: Absolute surface metrology by differencing spatially shifted maps from a phase-shifting interferometer. *Opt. Lett.* **35**(14), 2346–2348 (2010)
- Soons, J.A., Griesmann, U.: Absolute interferometric tests of spherical surfaces based on rotational and translational shears. *Proc. SPIE* **8493**, 84930G (2012)
- Su, D., Miao, E., Sui, Y., Yang, H.: Absolute surface figure testing by shift-rotation method using Zernike polynomials. *Opt. Lett.* **37**, 3198–3200 (2012)
- Weibo, W., Mengqian, Z., Siwen, Y., Zhigang, F., Jiubin, T.: Absolute spherical surface metrology by differencing rotation maps. *Appl. Opt.* **54**(20), 6186–6189 (2015)

- Weibo, W., Pengfei, L., Yaolong, X., Jiubin, T., Jian, L.: Error correction for rotationally asymmetric surface deviation testing based on rotational shears. *Appl. Opt.* **55**(26), 7428–7433 (2016)
- Song, W., Wu, F., Hou, X.: Method to test rotationally asymmetric surface deviation with high accuracy. *Appl. Opt.* **51**, 5567–5572 (2012)
- Evans, C.J., Kestner, R.N.: Test optics error removal. *Appl. Opt.* **35**(7), 1015–1021 (1996)
- Hyug-Gyo, R., Yun-Woo, L.: Azimuthal position error correction algorithm for absolute test of large optical surfaces. *Opt. Express* **14**(20), 9169–9177 (2006)

Submit your manuscript to a SpringerOpen® journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at ► [springeropen.com](http://springeropen.com)