Journal of the European Optical Society - Rapid Publications 1, 06008 (2006) www.jeos.org

Illustrations of optical vortices in three dimensions

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Optical vortices (phase singularities) arise from interference and are threads of darkness embedded within light fields. Although usually visualised in terms of their points of intersection with an observation plane, appreciation of their true form requires a view in three dimensions. Through numerical simulation we re-examine three situations where optical vortex lines evolve as an additional parameter is varied. Our specific examples are: the addition of a fourth plane wave of varying amplitude into a superposition of three plane waves, the increase of height of a non-integer spiral phase step, and finally the perturbation that creates, and then dissolves, a vortex link in a specific combination of Laguerre-Gauss modes. [DOI: 10.2971/jeos.2006.06008]

Keywords: Optical vortex, reconnections, interference

1 Introduction

When light waves propagating in different directions overlap, interference results in a complicated spatial variation in intensity. Provided that no single wave in the superposition is more intense than the sum of the others, destructive interference leads to places of complete darkness. Generically – that is, with no particular symmetry – this complete destructive interference occurs along lines. These lines are called optical vortices, nodal lines, phase singularities or wave dislocations [1, 2] and for monochromatic fields these lines are stationary in time.

Laser speckle patterns are laced with these optical vortices. A spatially coherent laser beam randomly reflected or scattered can be represented by a superposition of many plane waves, whose interference produces the familiar speckles of bright and dark spots on a screen [3]. Upon closer inspection, the dark spots are points of complete darkness around which the optical phase of the light varies by $\pm 2\pi$. These points of darkness are intersections of the vortex line with the viewing plane. The 3D nature of these vortices is revealed as this plane is translated along the optical axis, mapping out the three-dimensional vortex line structure.

An optical vortex can be located numerically or experimentally by considering a closed line integral of phase around each point within an observation plane. This integral is $\pm 2\pi$ when a vortex is enclosed, and is zero otherwise (for simplicity, we assume here that all vortices can be resolved individually). This azimuthal change in phase results in circulation of the Poynting vector, so phase singularities are vortices of optical energy flow [4, 5]. The study of vortex points in a single plane has led to a description of their behaviour in terms of birth and annihilation [6], where, on moving from one cross section to the next, a pair of neighbouring oppositely-signed vortices seems to appear or disappear. However, with the 3D structure recognised, these events are simply seen as static positions where a single curved vortex line changes its direction with respect to optical axis, in the form of a hairpin [7]. Such hairpins in vortex structure occur always in regions of low light intensity and can therefore be difficult to locate precisely.

Optical vortex lines can themselves evolve and undergo topological events. As a fourth parameter is varied (beyond the three spatial coordinates), small loops may nucleate or shrink to nothing, and lines may reconnect [5, 8]. These topological events are analogous to the behaviour of quantized vortices in other physical systems [9].

Within this work we calculate the interference pattern between the constituent waves on a 256×256 lateral grid for 256 different planes, producing a cube of interference data. Examining the phase for each 2×2 grid of pixels allows the vortex locations to be identified by phase unwrapping. Within any single cross section the positions of vortex hairpins can be ambiguous and consequently the interference cube is examined along the three cartesian directions. Once the list of vortex positions is generated, the vortex lines are plotted using the ray-tracing software, POVray [10]. Repeating the calculations as one or more parameters are varied allows an animated sequence to be constructed. On a standard desk-top computer, calculation times for each of these animated sequences amounts to a few hours.

We reconsider three simple physical situations in which op-

Received July 31, 2006; published August 8, 2006 ISSN 1990-2573

tical vortex lines evolve under the variation of a parameter. By numerically calculating and illustrating the evolution of the vortex line geometry in 3D, we hope to elucidate important attributes of optical vortex behaviour which may appear without logic in 2D. The specific problems we examine are: the addition of a fourth wave of varying amplitude into a superposition of three plane waves, the vortex structure in field propagating from a spiral phase plate as the height of the step varies, and the creation and destruction of a vortex link in a specific superposition of Laguerre-Gauss beams as the amplitude of the perturbing beam changes.

2 FOUR WAVE INTERFERENCE

For two interfering plane waves, complete destructive interference can only occur when the two wave amplitudes are equal, resulting in interference fringes which, in three dimensions, are planes of zero intensity. Otherwise, a prerequisite for complete destructive interference is that none of the interfering waves has an amplitude which exceeds the sum of the others. With three plane waves, the vortex lines are always straight and parallel, with a direction for which each wavevector has the same propagation component.

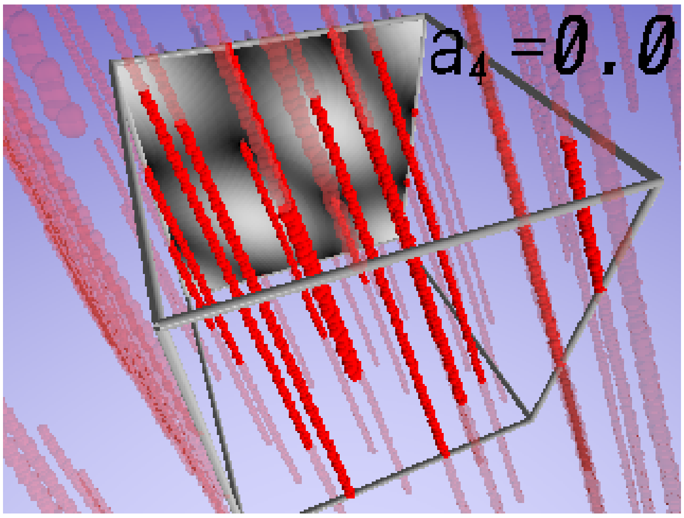

FIG. 1 Vortex structure arising from four plane wave interference. The transverse wavevectors have been chosen to lie on a square lattice, so by the Talbot effect, the wave field is periodic in 3D [11]. The cube frame shows the repeating cell. The associated movie shows the smooth deformation of this structure as the amplitude of a fourth wave, a_4, is increased from zero to the sum of the other amplitudes (where $a_1 = a_2 = a_3 = 1$) (see Fig1.mov, 3.6Mb).

If a fourth wave is added, with an amplitude increasing from zero, the straight vortex lines become helices [2]. As the amplitude increases, the vortex helices approach, touch, and undergo a reconnection, so the infinite vortex lines become closed vortex rings. The rings shrink to points and vanish as the amplitude of the fourth wave continues to increase beyond the sum of the other three [11]. Thus, as a single parameter is varied in the interference of four plane waves, both types of topological event [8,12] (loop nucleation/vanishing, reconnections) occur.

Figure 1 is an animated sequence showing the four-wave interference pattern with three wave amplitudes fixed at $a_1 = a_2 = a_3 = 1$ while the fourth wave amplitude, a_4, increases from zero to equal the sum of the other three. Although three amplitudes have been fixed, the adjustment of the fourth amplitude provides all possible topological configurations of four wave interference.

3 SPIRAL PHASE PLATE

For studies of optical orbital angular momentum, it is common to create a single optical vortex line along the beam axis. A common method is to use a spiral phase plate, whose optical thickness increases with azimuthal angle, so that a normally incident, phase flat wave of the correct wavelength acquires a phase factor $\exp(i\ell\phi)$ and an associated orbital angular momentum of $\ell\hbar$ per photon. There are practical difficulties in matching the discontinuity height exactly to the illumination wavelength, so that in experimental realizations, the azimuthal phase change ℓ is not an integer multiple of 2π (similar to the dynamical 'snake' instability in nonlinear waves [9]).

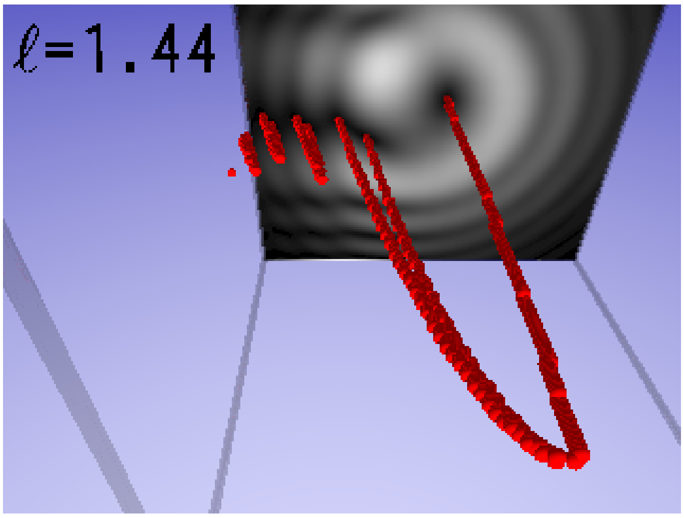

FIG. 2 The vortex structure of a beam after passing through a spiral phase plate of step height $\ell\lambda$. The associated movie shows how this 3D structure changes as the step height is increased from $\ell = 0$ to $\ell = 3$ (the greyscale plot represents the intensity after propagation of a small fraction of the Rayleigh range of the beam) (see Fig2.mov, 2.5Mb).

It was shown theoretically in Ref. [13] that when a plane wave illuminates a spiral phase plate, the discontinuity from half-integer ℓ leads to a radial sequence of vortices of alternating handedness along the original discontinuity. In the far field, this string of vortices is present whenever ℓ is not an integer. These predictions have been confirmed by experiment [14], in which plane wave illumination was approximated by a large-waist gaussian beam. Topologically, the only way that the vortex structure within a beam evolving from a non-integer phase step can acquire an infinite sequence of alternating-sign vortex points in the plane of the far field is that pairs of alternate-signed vortex points lie on a vortex 'hairpin'. The turning point of each hairpin extends towards the near field of the phase plate, and is is closest to the phase plate for half integer values of ℓ. The total number of hairpins and their closeness to the phase plate depends upon the size of the illuminating gaussian beam.

Figure 2 is an animated sequence of the vortex structure as the step height of the spiral phase plate is varied from $\ell = 0$ to $\ell = 3$.

4 VORTEX LINKS

In real beams of finite extent, the topology of optical vortices can be quite intricate, but may still be controlled. It was proposed in Ref. [5] that certain combinations of four beams would have vortices that are linked or knotted; in principle, any torus knot or link is possible with this construction. The superposition in the knot construction involves three coaxial, copropagating beams carrying equal, nonzero angular momentum (e.g. Bessel beams [5], polynomial beams [12] or Laguerre-Gauss beams [15]), with particular amplitudes. These are perturbed by a further beam with no vortices (e.g. a J_0 Bessel beam, plane wave, or gaussian beam).

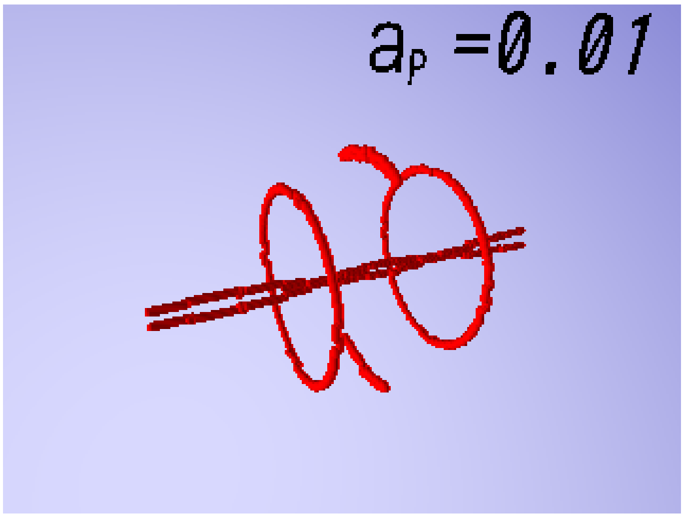

FIG. 3 Vortex configuration of Laguerre-Gauss beam superposition giving a vortex link [15]. The figure depicts the vortex structure of the initial field which is perturbed to create a vortex link. The associated movie shows the structure undergoing a topological transition as the perturbation, a_p is increased from 0 to 0.75; the link is created through reconnection when $a_p = 0.3$, and dissolves through further reconnections when $a_p = 0.5$; the transverse intensity is scanned for fixed $a_p = 0.35$ (see Fig3.mov, 3.6Mb).

In paraxial beams, the unperturbed vortex configuration consists of a system of unlinked, axisymmetric rings, one of which has zero strength (the phase does not change in a loop around it). As the amplitude of the perturbation increases, these rings deform, crescent-shaped loops nucleate from the zero strength loop, and the central vortex line unfolds to an n-stranded helix [12]. At some lower critical value of the perturbation amplitude, the vortex loops reconnect to give the appropriate torus knot or link. At an upper critical value, there are further reconnections between the knot or link and the axial helix, dissolving the knot. This controlled method of knot and link creation was implemented experimentally in Refs. [15, 16], in which a Hopf link ((2,2) torus link) and trefoil knot ((3,2) torus knot) were synthesized.

The process in which the trefoil link forms and then dissolves via reconnections, as the amplitude of the perturbing beam is increased, is illustrated in Figure 3.

5 DISCUSSION

The dynamical evolution of optical vortex points in a 2D observation plane can be better understood as a pattern of lines embedded within the volume of a light beam. For monochromatic fields, the vortex lines are stationary in space, so possess no temporal dynamics. We have illustrated here simple examples of how the vortex lines do evolve under the change of an additional parameter (in the first and third examples, this is the amplitude of an additional wave in the superposition; in the second, a change of an initial field profile). Other examples of vortex line evolution have been discussed, such as Refs. [17, 18], in which vortices in diffraction patterns approach the characteristic configuration of the canonical diffraction catastrophes, as the dimension of the diffraction aperture approaches infinity.

Since the vortices are zeros in the complex scalar amplitude, the dependence of their position with respect to this parameter is highly nonlinear, just as the vortex lines are not straight upon propagation [6]. The most significant events in the evolution of vortex lines are loop nucleation/vanishing and reconnection; these events occur stably as a parameter is varied [8] (they are codimension 4); with dependence on further parameters, more complicated topological interactions may occur [17]. As our movie illustrations demonstrate, complicated optical vortex topologies arise (such as links) solely through these two types of event.

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